

Source Information Transmission over MIMO Systems with Transmitter Side Information

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Abstract—We consider strategies for the lossy transmission of a zero-mean memoryless Gaussian source over a 2×2 Rayleigh faded MIMO system with transmitter side information. The first strategy uses repetition coding, where the same symbol is duplicated over the two transmit antennas in two consecutive time slots. The second strategy employs the Alamouti scheme, while the third strategy is based on spatial multiplexing. A mean squared error distortion measure is assumed. Since this distortion results, at the receiver, in a continuous random variable, both its expected value and statistical distribution are used for evaluating the performance of each strategy. It is shown that the spatial multiplexing strategy achieves the lowest expected distortion and also that the utilization of the Alamouti strategy brings only a slight decrease in performance, but has the advantage of a lower realization complexity. Considering the statistical distributions of distortion, it is observed that increasing the signal-to-noise ratio results in higher probabilities of achieving the expected distortion and to less variable values of distortion. Due to the characteristics of the respective distributions of distortion, both Alamouti and spatial multiplexing strategies are capable of achieving low distortion with high probability while, at the same probability, the repetition strategy achieves significantly higher distortion.

I. INTRODUCTION

We consider the lossy transmission of Gaussian source information over 2×2 Multiple-Input Multiple-Output (MIMO) systems with Rayleigh fading, assuming the knowledge of Channel State Information (CSI) at the transmitter (also called transmitter side information).

In [1], we considered Gaussian source transmission over 2×2 MIMO systems when CSI is not available at the transmitter. In that work, we developed and compared several strategies based on techniques such as Repetition coding (REP) [2], Time Sharing (TS), the Alamouti scheme (ALM) [3] and Spatial Multiplexing (SM) [2]. Since the transmitter did not have knowledge of CSI, it did not know the instantaneous rate supported by the channel, i.e. its capacity, and hence it was not able to adapt the source coding rate to the channel conditions to ensure the decoding of the information

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This research has been supported by the California Micro Program, Applied Signal Technology, Cisco, Dolby Labs, Inc., Sony-Ericsson, by Qualcomm, Inc, by NSF Grant Nos. CCF-0429884, CNS-0435527, CCF-0728646, and by the European Community under Seventh Framework Programme grant agreement ICT OPTIMIX n°INFSO-ICT-214625.

at the receiver with an arbitrarily small probability of error. Instead, it encoded and transmitted the source information using a constant rate, chosen to achieve a selected *outage probability*. When the channel did not support the transmission of information at the chosen coding rate, data were lost and the system experienced an *outage*.

In this work, we want to reconsider these strategies under the assumption of perfect CSI at the transmitter. In this case, the transmitter is able to follow the (slow) variations of the channel by adapting the source coding rate to the instantaneous capacity, since it is aware of the particular channel realization in every time instant. In such a situation there is no notion of outage since the source rate is always adapted to achieve the instantaneous channel capacity [4].

This observation has a direct impact on the usefulness of the TS strategies in our current scenario. These strategies employ a time sharing approach to the two transmit antennas to create two independent channels from our MIMO system [1]. These independent channels are then used to provide path diversity by transmitting multiple description representations of the source over them. However, path diversity is useful only if the channels are unreliable, i.e. if they suffer outages, which is not our current case. For this reason, in this paper we do not consider the TS strategies.

The remaining three strategies are described in the following sections, where we also evaluate their performance using a mean squared error fidelity criterion. In particular, since the distortion at the receiver results in a continuous random variable, we consider both its expected value and statistical distribution, i.e. its cumulative distribution function (CDF). Finally, a comparison of the performance of the different strategies is made.

II. STRATEGIES FOR INFORMATION TRANSMISSION

A. Assumptions and preliminaries

We consider a 2×2 MIMO system characterized by the channel matrix H , having the form

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

Each entry h_{ij} of the channel matrix H represents the gain of the channel between the j -th transmit antenna and i -th receive antenna. Each one of these channels is assumed to

be independent, random and with very slow Rayleigh fading. The h_{ij} are then i.i.d. complex Gaussian random variables with zero mean and unit variance, which remain constant over the transmission of a large number of symbols. Under these assumptions, the squared magnitude of the channel gains can be written as

$$|h_{ij}|^2 = \frac{1}{2}x_{ij}, \quad i, j = 1, 2 \quad (1)$$

where the x_{ij} are random variables distributed according to a chi-square distribution with 2 degrees of freedom [5]. Perfect CSI, i.e. knowledge of H , is assumed to be available both at transmitter and receiver.

The total transmitted power by the transmit antennas is constrained to P_t . If both transmit antennas are transmitting simultaneously, each antenna will transmit with equal power $P_t/2$, while, if only one antenna is transmitting at a given time, it can make use of full transmit power P_t . The noise at the receiver is i.i.d. AWGN noise, with the same average power N at each receive antenna.

The source is assumed to be a zero-mean memoryless Gaussian source with a variance normalized to unity. The system bandwidth is also assumed to be normalized to unity.

In the following, we will denote with $\bar{\gamma}$ the ratio P_t/N and with $\Gamma(z)$ and $\Gamma(a, z)$, respectively, the gamma function and the incomplete gamma function [5]. We will also denote with χ_k^2 the distribution of a chi-square random variable with k degrees of freedom, with $F_\chi^{(k)}(z)$ its CDF and with

$$f_\chi^{(k)}(z) = \frac{1}{\Gamma(\frac{k}{2})2^{\frac{k}{2}}} z^{\frac{k-2}{2}} e^{-\frac{z}{2}}$$

its probability density function (PDF) [5].

B. REP strategy

In the REP strategy [1], the same symbol S_1 is transmitted over the two transmit antennas in two consecutive time slots. In each time slot, only one of the two transmit antennas is used for transmission, while the other antenna is turned off.

Thus, in the first time slot S_1 is transmitted on the first transmit antenna and it is observed by the receiver through the two channels with gains h_{11} and h_{21} . In the second time slot, the same symbol S_1 is transmitted on the second transmit antenna and it is observed by the receiver through the two channels with gains h_{12} and h_{22} . A Maximal Ratio Combiner (MRC) [6] is then used at the receiver to optimally combine the four signals received by the two receive antennas in the two different time slots.

Since transmitter side information does not increase capacity unless transmitted power is also adapted [6], the capacity of this strategy in a given fading realization has the same expression as in [1] and results

$$C = \frac{1}{2} \log_2 \left(1 + \bar{\gamma} \sum_{i,j=1}^2 |h_{ij}|^2 \right)$$

which can be rewritten using Eq. (1) as

$$C = \frac{1}{2} \log_2 \left(1 + \frac{\bar{\gamma}}{2} \sum_{i,j=1}^2 x_{ij} \right) = \frac{1}{2} \log_2 \left(1 + \frac{\bar{\gamma}}{2} x_s \right)$$

where $\sum_{i,j=1}^2 x_{ij} = x_s \sim \chi_8^2$ [5].

Since the transmitter has CSI knowledge, in every time instant the source coding rate R_{REP} can be adapted to achieve the instantaneous capacity C . The distortion D_r observed at the receiver is then [7]

$$D_r = 2^{-2R_{REP}} = \frac{1}{1 + \frac{\bar{\gamma}}{2}x_s}$$

which is a continuous random variable. Its expected value is

$$\begin{aligned} D_{REP} = \mathbb{E}[D_r] &= \int_0^{+\infty} \frac{1}{1 + \frac{\bar{\gamma}}{2}z} f_\chi^{(8)}(z) dz \\ &= \frac{1}{48} \int_0^{+\infty} \frac{z^3}{2 + \bar{\gamma}z} e^{-\frac{z}{2}} dz \end{aligned}$$

which yields

$$D_{REP} = \frac{1}{6} \cdot \frac{\bar{\gamma} - \bar{\gamma}^2 + 2\bar{\gamma}^3 - e^{\frac{1}{\bar{\gamma}}}\Gamma\left(0, \frac{1}{\bar{\gamma}}\right)}{\bar{\gamma}^4}$$

The CDF $F_{REP}(d)$ of the distortion at the receiver can be derived as

$$\begin{aligned} F_{REP}(d) = Pr\{D_r < d\} &= Pr\left\{x_s > \frac{2-2d}{\bar{\gamma}d}\right\} \\ &= 1 - F_\chi^{(8)}\left(\frac{2-2d}{\bar{\gamma}d}\right) \end{aligned}$$

C. ALM strategy

This strategy [1] employs the Alamouti scheme to obtain two independent channels from the MIMO system. These two channels are then used for the transmission of a single description representation of the source, after demultiplexing it into two half-rate substreams. The capacity for the ALM strategy is given by [1]

$$C = \log_2 \left(1 + \frac{\bar{\gamma}}{2} \sum_{i,j=1}^2 |h_{ij}|^2 \right)$$

that, from Eq. (1), can be expressed as

$$C = \log_2 \left(1 + \frac{\bar{\gamma}}{4} \sum_{i,j=1}^2 x_{ij} \right) = \log_2 \left(1 + \frac{\bar{\gamma}}{4} x_s \right)$$

where $\sum_{i,j=1}^2 x_{ij} = x_s \sim \chi_8^2$ [5].

Using transmitter side information, the source coding rate R_{ALM} can be adjusted to follow the variations of the capacity C . Thus, the distortion at the receiver is given by [7]

$$D_r = 2^{-2R_{ALM}} = \frac{1}{(1 + \frac{\bar{\gamma}}{4}x_s)^2}$$

Its expected value can be evaluated as

$$\begin{aligned} D_{ALM} = \mathbb{E}[D_r] &= \int_0^{+\infty} \frac{1}{(1 + \frac{\bar{\gamma}}{4}z)^2} f_\chi^{(8)}(z) dz \\ &= \frac{1}{6} \int_0^{+\infty} \frac{z^3}{(4 + \bar{\gamma}z)^2} e^{-\frac{z}{2}} dz \end{aligned}$$

which finally results in

$$D_{ALM} = \frac{2}{3} \cdot \frac{\bar{\gamma} [(\bar{\gamma} - 4)\bar{\gamma} - 4] + 4e^{\frac{2}{\bar{\gamma}}} (3\bar{\gamma} + 2)\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)}{\bar{\gamma}^5}$$

The CDF $F_{ALM}(d)$ of the distortion is

$$\begin{aligned} F_{ALM}(d) &= Pr\{D_r < d\} = Pr\left\{x_s > \frac{4 - 4\sqrt{d}}{\bar{\gamma}\sqrt{d}}\right\} \\ &= 1 - F_x^{(8)}\left(\frac{4 - 4\sqrt{d}}{\bar{\gamma}\sqrt{d}}\right) \end{aligned}$$

D. SM strategy

In the SM strategy [1], a single description of the source, i.e. a single symbol stream, is first demultiplexed and encoded into two separate and independent substreams. Each substream is then transmitted simultaneously over each transmit antenna and, at the receiver, an optimal joint maximum likelihood (ML) decoder is employed for retrieving the original symbol stream. The capacity of this strategy is given by [1]

$$C = \log_2 \det \left(I_2 + \frac{\bar{\gamma}}{2} HH^H \right) \quad (2)$$

where I_2 is the 2×2 identity matrix and H^H denotes the conjugate transpose of the channel matrix H .

To determine the expression of the expected distortion at the receiver for this strategy, we first need to introduce the characteristic function $\phi_C(z)$ of the capacity C , defined as [8]

$$\phi_C(z) = \mathbb{E}\left[e^{j2\pi Cz}\right] = K \det[U(z)]$$

where, for a 2×2 uncorrelated MIMO Rayleigh fading channel, $K = 1$ and $U(z)$ is a 2×2 matrix with ik -th elements given by [8]

$$u_{ik}(z) = \int_0^{+\infty} x^{i+k-2} e^{-x} \left(1 + \frac{\bar{\gamma}}{2}x\right)^{j \frac{2\pi z}{\ln 2}} dx \quad (3)$$

Since the source coding rate R_{SM} is adapted, in every time instant, to the capacity C , we can now express the expected distortion D_{SM} in terms of the function $\phi_C(z)$ as [7]

$$\begin{aligned} D_{SM} &= \mathbb{E}\left[2^{-2R_{SM}}\right] = \mathbb{E}\left[e^{-2C \ln 2}\right] = \phi_C\left(j \frac{\ln 2}{\pi}\right) \\ &= \det \left[U\left(j \frac{\ln 2}{\pi}\right) \right] = \hat{u}_{11}\hat{u}_{22} - \hat{u}_{12}\hat{u}_{21} \end{aligned} \quad (4)$$

where we defined $\hat{u}_{ik} = u_{ik}(j \ln 2/\pi)$. Developing the expression in Eq. (3), we get

$$\begin{aligned} \hat{u}_{11} &= 2 \cdot \frac{\bar{\gamma} - 2e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)}{\bar{\gamma}^2} \\ \hat{u}_{12} &= 4 \cdot \frac{(2 + \bar{\gamma})e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right) - \bar{\gamma}}{\bar{\gamma}^3} \\ \hat{u}_{21} &= \hat{u}_{12} \\ \hat{u}_{22} &= 4 \cdot \frac{\bar{\gamma}(2 + \bar{\gamma}) - 4(1 + \bar{\gamma})e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)}{\bar{\gamma}^4} \end{aligned}$$

and, after substituting these expressions into Eq. (4), gives the final expression of D_{SM} as

$$\begin{aligned} D_{SM} &= -\frac{16\left[\bar{\gamma} - (\bar{\gamma} + 2)e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)\right]^2}{\bar{\gamma}^6} \\ &+ \frac{8\left[\bar{\gamma} - 2e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)\right]\left[\bar{\gamma}(\bar{\gamma} + 2) - 4(\bar{\gamma} + 1)e^{\frac{2}{\bar{\gamma}}}\Gamma\left(0, \frac{2}{\bar{\gamma}}\right)\right]}{\bar{\gamma}^6} \end{aligned}$$

To derive the CDF $F_{SM}(d)$ of the distortion at the receiver, we first need to rewrite the expression of the capacity in Eq. (2) using the singular value decomposition of H , to obtain [9]

$$C = \log_2 \prod_{i=1}^2 \left(1 + \frac{\bar{\gamma}}{2}\lambda_i\right) = \log_2(x_1 x_2)$$

where

$$x_i = 1 + \frac{\bar{\gamma}}{2}\lambda_i, \quad i = 1, 2 \quad (5)$$

and the λ_i are the two ordered nonzero eigenvalues of the matrix HH^H . Since $\lambda_i > 0$ and $\lambda_1 \geq \lambda_2$, it is immediate to show that $x_i > 1$ and $x_1 \geq x_2$.

The distortion observed at the receiver is then [7]

$$D_r = 2^{-2R_{SM}} = \frac{1}{(x_1 x_2)^2}$$

and $F_{SM}(d)$ results

$$\begin{aligned} F_{SM}(d) &= Pr\{D_r < d\} = 1 - Pr\left\{x_1 x_2 < \frac{1}{\sqrt{d}}\right\} \\ &= 1 - F\left(\frac{1}{\sqrt{d}}\right) \end{aligned} \quad (6)$$

where $F(z) = Pr\{x_1 x_2 < z\}$.

We now define S as the set of points (x_1, x_2) such that

$$S = \left\{ (x_1, x_2) \in \mathbb{R}^2 : x_1 > 1; x_2 > 1; x_1 \geq x_2; x_1 x_2 < z \right\}$$

If X is the vector $X = (x_1, x_2)$ with joint PDF $f_X(x_1, x_2)$, $F(z)$ can then be evaluated as

$$\begin{aligned} F(z) &= \int_S f_X(x_1, x_2) dx_1 dx_2 \\ &= \frac{1}{2} \int_1^z \int_1^{\frac{z}{x_2}} f_X(x_1, x_2) dx_1 dx_2 \end{aligned} \quad (7)$$

The joint PDF of X can be expressed in terms of the joint PDF $f_\Lambda(\lambda_1, \lambda_2)$ of the vector of the eigenvalues $\Lambda = (\lambda_1, \lambda_2)$ as [10]

$$f_X(x_1, x_2) = \frac{f_\Lambda(\lambda_1, \lambda_2)}{\det(J)} = \frac{4}{\bar{\gamma}^2} f_\Lambda(\lambda_1, \lambda_2) \quad (8)$$

where J is the Jacobian matrix of X . Since from Eq. (5) we have

$$\lambda_i = \frac{2}{\bar{\gamma}}(x_i - 1), \quad i = 1, 2$$

we can rewrite Eq. (8) as

$$f_X(x_1, x_2) = \frac{4}{\bar{\gamma}^2} f_\Lambda\left(\frac{2}{\bar{\gamma}}(x_1 - 1), \frac{2}{\bar{\gamma}}(x_2 - 1)\right) \quad (9)$$

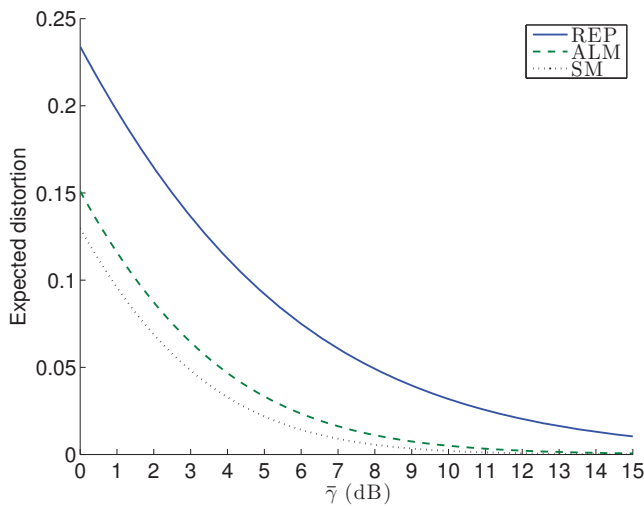


Fig. 1: Expected distortion as a function of $\bar{\gamma}$ for the different strategies.

The joint PDF of the eigenvalues for a 2×2 system with uncorrelated fading between each antenna element can be obtained from [11] and results

$$f_{\Lambda}(\lambda_1, \lambda_2) = e^{-\lambda_1} e^{-\lambda_2} (\lambda_1 - \lambda_2)^2$$

which, substituting into Eq. (9), yields

$$f_X(x_1, x_2) = \frac{16}{\bar{\gamma}^4} e^{\frac{4}{\bar{\gamma}}} e^{-\frac{2}{\bar{\gamma}}(x_1+x_2)} (x_1 - x_2)^2$$

Substituting this last expression into Eq. (7) gives

$$F(z) = \frac{8}{\bar{\gamma}^4} e^{\frac{4}{\bar{\gamma}}} \int_1^z e^{-\frac{2}{\bar{\gamma}}x_2} \int_1^{\frac{z}{x_2}} e^{-\frac{2}{\bar{\gamma}}x_1} (x_1 - x_2)^2 dx_1 dx_2$$

which can be finally written as

$$F(z) = \frac{2}{\bar{\gamma}^3} e^{\frac{4}{\bar{\gamma}}} \int_1^z e^{-\frac{2}{\bar{\gamma}}x_2} \cdot \left\{ e^{-\frac{2}{\bar{\gamma}}} \left[\bar{\gamma}^2 - 2\bar{\gamma}(x_2 - 1) + 2(x_2 - 1)^2 \right] - e^{-\frac{2}{\bar{\gamma}}\frac{z}{x_2}} \left[\bar{\gamma}^2 + 2\bar{\gamma}\left(\frac{z}{x_2} - x_2\right) + 2\left(\frac{z}{x_2} - x_2\right)^2 \right] \right\} dx_2$$

By using this expression into Eq. (6), it is then possible to evaluate the CDF of distortion for the SM strategy.

III. DISCUSSION

Fig. 1 plots the expected distortion as a function of $\bar{\gamma}$ for the different strategies. As can be seen, the lowest distortion is achieved with the SM strategy at all values of $\bar{\gamma}$. This performance, however, comes at the expense of realization complexity, due to the presence of the joint ML decoder at the receiver [1]. Similarly to the case of CSI at the receiver only of [1], if the ALM strategy is employed to reduce this complexity, only a small decrease in performance is observed, especially at high values of $\bar{\gamma}$. Significantly higher distortion is achieved with the REP strategy.

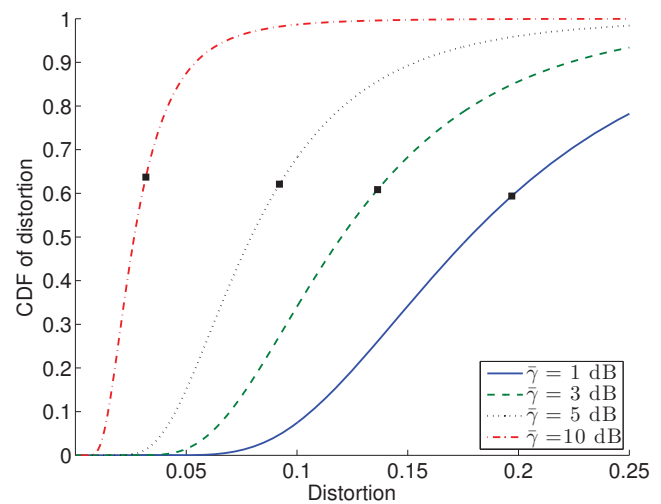


Fig. 2: CDF of distortion for REP strategy with different values of $\bar{\gamma}$.

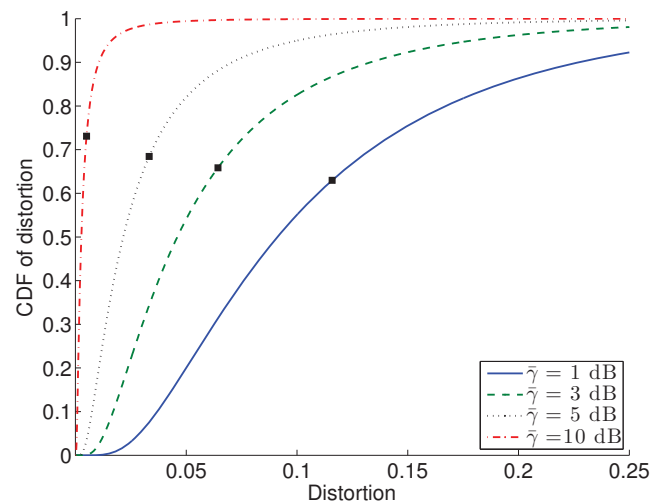


Fig. 3: CDF of distortion for ALM strategy with different values of $\bar{\gamma}$.

Figs. 2, 3 and 4 plot the CDF of the distortion for different values of $\bar{\gamma}$ respectively for REP, ALM and SM strategies. The square markers in the plots represent the value of the expected distortion for the respective value of $\bar{\gamma}$. Interestingly, an increase in the value of $\bar{\gamma}$ not only improves the value of expected distortion for every strategy (as it appears evident also from Fig. 1), but also improves the probability $p_e(\bar{\gamma})$ of achieving that distortion. The values of $p_e(\bar{\gamma})$ for the different strategies and for different values of $\bar{\gamma}$ are reported in Table I. It can be observed that, given the same increase of $\bar{\gamma}$, the increase in $p_e(\bar{\gamma})$ in the REP strategy is significantly lower than the increase in $p_e(\bar{\gamma})$ for the remaining two strategies. For example, if $\bar{\gamma}$ increases from 1 dB to 10 dB, $p_e(\bar{\gamma})$ in the REP strategy increases of about 0.04, while in ALM and SM it increases of about 0.10 and

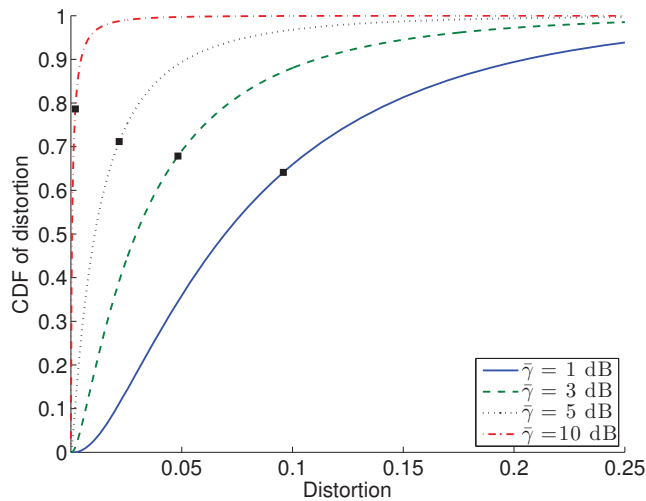


Fig. 4: CDF of distortion for SM strategy with different values of $\bar{\gamma}$.

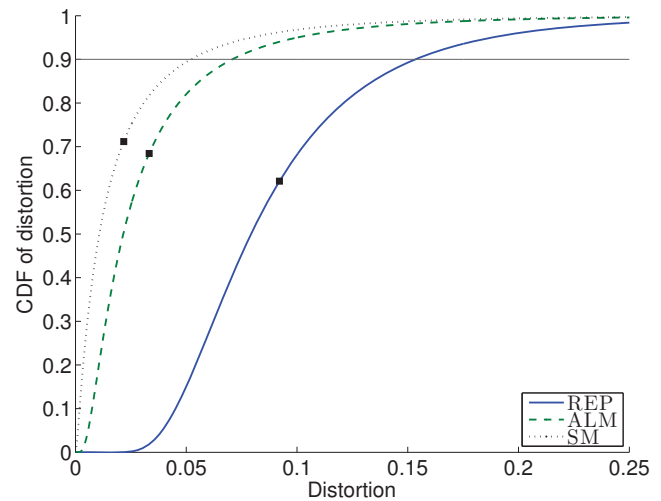


Fig. 5: CDF of the distortion at the receiver for the different strategies at a fixed $\bar{\gamma}$ of 5 dB.

TABLE I

PROBABILITY OF ACHIEVING EXPECTED DISTORTION FOR THE VARIOUS STRATEGIES WITH DIFFERENT VALUES OF $\bar{\gamma}$.

	$p_e(\bar{\gamma})$			
	$\bar{\gamma} = 1$ dB	$\bar{\gamma} = 3$ dB	$\bar{\gamma} = 5$ dB	$\bar{\gamma} = 10$ dB
ALM	0.630	0.658	0.684	0.731
REP	0.594	0.608	0.621	0.637
SM	0.641	0.678	0.712	0.787

0.14, respectively. Moreover, returning to Figs. 2, 3 and 4, an increase of $\bar{\gamma}$ causes also an increase in the slope of the CDF for all strategies, suggesting that the values of distortion become less variable as $\bar{\gamma}$ increases.

A comparison of the CDF of the distortion for the various strategies at a fixed $\bar{\gamma}$ of 5 dB is reported in Fig. 5. Both ALM and SM strategies have similar and very steep CDFs, which means that it is possible with these strategies to achieve low values of distortion with high probability. For example, with a probability of 0.9, SM achieves a distortion approximately equal to 0.05, while ALM achieves a distortion approximately equal to 0.07. The REP strategy has a much less steep CDF than the other two strategies and indeed with a probability of 0.9 it achieves a significantly higher distortion, approximately equal to 0.15.

IV. CONCLUSIONS

We considered three strategies for the transmission of a Gaussian source over a 2×2 MIMO system with transmitter side information. These strategies are based on repetition coding, the Alamouti scheme and spatial multiplexing. First, we evaluated the expected value of the distortion at the receiver and showed that the best performance is achieved with the SM strategy, but at the expense of high realization complexity. We also observed that employing the ALM

strategy brings only a slight decrease in performance, but can effectively reduce this complexity. Then, we considered the statistical distributions of the distortion at the receiver. We showed that an increase in the signal-to-noise ratio brings, for every strategy, higher probabilities of achieving the expected distortion and less variation of distortion. Finally, by comparing the distributions we observed that both ALM and SM strategies can achieve low values of distortion with high probability, while at the same probability the REP strategy achieves significantly higher distortions.

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