Source Coding Diversity and Multiplexing Strategies for a 2X2 MIMO System

Marco Zoffoli*
Dept. of Electrical and Computer Eng.
University of California, Santa Barbara

Jerry D. Gibson†
Dept. of Electrical and Computer Eng.
University of California, Santa Barbara
Email: gibson@ece.ucsb.edu

Marco Chiani
DEIS
University of Bologna
47023 Cesena, Italy

***Abstract*—We consider strategies for the lossy transmission of a zero mean Gaussian source over multiple channels. In one strategy, we employ single description coding of the source and duplicate this description over two independent channels. We also consider optimal, no excess joint rate, and no excess marginal rate multiple description coding over two independent channels. These strategies are compared to the traditional approach where a single description is sent over a single channel. The performance measure used for comparison is expected distortion at the receiver, evaluated as a function of the outage probability. We also consider the transmission of a full rate single description over a standard 2X2 MIMO channel using spatial multiplexing and a time-sharing approach to using MD coding over a 2X2 MIMO channel.**

I. Introduction

In a 2X2 MIMO system we have a total of 4 channels, one for each transmit/receive antenna pair [1]. However, to begin the discussion we assume a simplified system where the cross channels are not present, so that we are considering two parallel, independent channels. Four different strategies are used for the transmission of a zero mean gaussian source over this system, employing single and multiple description coding.

The first strategy is called duplicate single description coding (DSD). It consists in representing the source with a single description code of rate R_{DSD} and duplicating this description over the two transmit antennas.

The remaining strategies employ different types of multiple description coding, all having the same joint description rate R_{MD}. These strategies are, namely, no excess marginal rate (MD-NMR), no excess joint rate (MD-NJR) and optimal MD coding (MD-OPT) [2]. We also consider the traditional approach (SD), where a single description code of rate R_{SD} is sent over a single channel.

We compare these strategies using mean squared error distortion at the receiver evaluated as a function of outage probability, defined as the probability of having one channel in outage.

*Currently visiting from University of Bologna.
†Corresponding author.
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Finally, we consider two MIMO strategies, one using spatial multiplexing and a single description source code and the other using a time-sharing approach and the MD codes.

II. Parallel channel strategies

A. Assumptions and preliminaries

We begin our development by considering the special case of parallel, independent channels.

All the variances and bandwidths are assumed to be normalized to unity. Both channels are assumed to be random, with very slow Rayleigh fading and are represented by complex gains h_1 and h_2. Thus, the h_i are i.i.d. zero mean complex Gaussian random variables which remain constant over the transmission of a large number of symbols.

Channel state information (CSI) is assumed not to be available at the transmitter, i.e. the transmitter does not have any knowledge of the h_i except for their statistical distribution.

Each antenna transmits with equal power $\hat{P}/2$, such that the total transmitted power is equal to $\hat{P}$. The average power received by each receive antenna is equal to $P/2$. The noise at the receiver is assumed to be i.i.d. AWGN noise, with the same average power N at each receive antenna.

The average signal to noise ratio (SNR) at the i-th receive antenna is then equal to

$$\bar{\gamma_i} = \frac{P}{2N} = \frac{\bar{\gamma}}{2}$$

where $\bar{\gamma} = P/N$, and the instantaneous SNR is given by [3]

$$\gamma_i = \|h_i\|^2 \bar{\gamma}_i = \|h_i\|^2 \frac{\bar{\gamma}}{2} \tag{1}$$

The instantaneous capacity C_i of i-th channel is thus [1]

$$C_i = \log_2 \Big(1 + \gamma_i\Big) = \log_2 \Big(1 + \|h_i\|^2 \frac{\bar{\gamma}}{2}\Big)$$

which results in a random variable due to its dependence on h_i.

Given a design parameter called *outage probability* (P_{out}), the value of capacity that is achieved on the i-th channel with probability equal to $1 - P_{out}$ is called *outage capacity* $C_{out}^{(i)}$ [4]

$$\begin{aligned} P_{out} &= Pr\{C_i < C_{out}^{(i)}\} \qquad (2) \\ &= Pr\Big\{ \log_2 \Big(1 + \|h_i\|^2 \frac{\bar{\gamma}}{2}\Big) < C_{out}^{(i)} \Big\} \end{aligned}$$

In our case, since the h_i are identically distributed, the outage capacities of the two channels for the same P_{out} are identical

$$C_{out}^{(1)} = C_{out}^{(2)} = C_{out} \quad (3)$$

and

$$P_{out} = Pr\{C_i < C_{out}\}$$

Given a value for P_{out}, each antenna will then transmit at a constant rate equal to C_{out} [5]. P_{out} represents the probability of having a channel in *outage*, i.e. the received symbols from the respective channel cannot be decoded with probability 1, simply because its capacity is lower than the transmitted rate.

If we write C_{out} as [1]

$$C_{out} = \log_2(1 + \gamma_{min}) \quad (4)$$

from Equations (2), (3) and (1) we get

$$P_{out} = Pr\{\gamma_i < \gamma_{min}\}$$

Thus, γ_{min} is the minimum SNR required at one receive antenna for having the corresponding channel not in outage [5]. Since the h_i are Gaussian distributed, it results that [6]

$$P_{out} = 1 - e^{-2\frac{\gamma_{min}}{\bar{\gamma}}}$$

and

$$\gamma_{min} = -\frac{\bar{\gamma}\log(1 - P_{out})}{2} \quad (5)$$

B. Duplicate Single Description

In this strategy, the same single description code of rate R_{DSD} is transmitted over the two independent channels.

Given P_{out}, the transmitted rate on each channel is given by Equations (4) and (5)

$$R_{DSD} = \log_2\left[1 - \frac{\bar{\gamma}\log(1 - P_{out})}{2}\right]$$

In the case of either no outage on both channels or outage on only one of the two channels, the receiver will observe a distortion equal to [7]

$$D_1 = 2^{-2R_{DSD}} \quad (6)$$

When both links are in outage the distortion observed by the receiver is 1.

The expected distortion is then given by

$$\begin{aligned} D &= (1 - P_{out})^2 D_1 + 2P_{out}(1 - P_{out})D_1 + P_{out}^2 \\ &= (1 - P_{out}^2)D_1 + P_{out}^2 \end{aligned}$$

C. Multiple Description

In this strategy we use multiple description coding to obtain two different descriptions of the source, which are independently sent over the two channels.

The MD encoder has joint rate equal to R_{MD}, so each description has rate $R_{MD}/2$. Similarly to the DSD case, the transmitted rate on each channel is chosen to be equal to the outage capacity at a given P_{out}

$$\frac{R_{MD}}{2} = \log_2\left[1 - \frac{\bar{\gamma}\log(1 - P_{out})}{2}\right]$$

When both channels are not in outage, the MD decoder can reconstruct the source from both descriptions, achieving a distortion equal to D_0. If only one of the two channels is available, the MD decoder can reconstruct the source from the only description received, achieving a higher distortion D_1.

The expected distortion has the following expression

$$D = (1 - P_{out})^2 D_0 + 2P_{out}(1 - P_{out})D_1 + P_{out}^2 \quad (7)$$

The relations between D_0, D_1 and R_{MD} can be obtained from the following proposition [8].

Proposition 1. *Let an i.i.d. Gaussian source with unit variance be described by two descriptions both of which have rate $R_{MD}/2$. The distortions D_1 and D_0 corresponding to observations of one or both descriptions. The achievable distortion region for a fixed rate $R_{MD}/2$ is described by:*

$$\begin{aligned} D_0 &\geq 2^{-2R_{MD}} \\ D_1 &\geq 2^{-R_{MD}} \\ \Big(D_0, D_1\Big) &\geq \left(a, \frac{1+a}{2} - \frac{1-a}{2}\sqrt{1 - \frac{2^{-2R_{MD}}}{a}}\right) \end{aligned}$$

for $a \in \left[2^{-2R_{MD}}, \frac{2^{-R_{MD}}}{2-2^{-R_{MD}}}\right]$.

From Proposition 1 we can obtain three different types of MD coders [2], each with different performance.

1) **No Excess Marginal Rate (MD-NMR).** In this type of MD coder, the two individual descriptions are chosen to be rate distortion optimal, with distortion

$$D_1 = 2^{-R_{MD}} \quad (8)$$

From Proposition 1 we get the following lower bound on distortion D_0

$$D_0 \geq \frac{2^{-R_{MD}}}{2 - 2^{-R_{MD}}} \quad (9)$$

Using these expressions in Equation (7), we get the desired results.

2) **No Excess Joint Rate (MD-NJR).** In this type of MD coder the joint description is rate distortion optimal, with distortion

$$D_0 = 2^{-2R_{MD}} \quad (10)$$

The lower bound on distortion D_1 can be obtained again from Proposition 1 and results

$$D_1 \geq \frac{1}{2}\left(1 + 2^{-2R_{MD}}\right) \quad (11)$$

Substituting these expressions into Equation (7) we get the results.

3) **Optimal MD coding (MD-OPT).** In this case the MD coder appropriately chooses the values of D_0 and D_1 to minimize the expected distortion D. From Equation (7)

and Proposition 1 we can write D as

$$D = (1 - P_{out})^2 a + 2P_{out}(1 - P_{out})\left(\frac{1+a}{2} - \frac{1-a}{2}\sqrt{1 - \frac{2^{-2R_{MD}}}{a}}\right) + P_{out}^2 \quad (12)$$

Given P_{out}, the MD-OPT coder chooses the value of a that minimizes D in Equation (12).

D. Single Description

Here we consider the traditional strategy in which a single description code of rate R_{SD} is transmitted over a single link. In order to make a fair comparison, we assume that the transmit antenna transmits with power $\hat{P}$, equal to the total transmitted power of the previous cases. Under this assumption, the average received power from the single antenna is P and the average received SNR is equal to $\bar{\gamma}$. The instantaneous channel capacity is then given by [1]

$$C = \log_2(1 + \|h\|^2\bar{\gamma})$$

Individual channel outage probability can be obtained similarly to before and results [5]

$$P_{out} = 1 - e^{-\frac{\gamma_{min}}{\bar{\gamma}}}$$

and [5]

$$\gamma_{min} = -\bar{\gamma}\log(1 - P_{out})$$

Given P_{out}, the transmitted rate R_{SD} is then [5]

$$R_{SD} = \log_2\left[1 - \bar{\gamma}\log(1 - P_{out})\right]$$

and distortion D_1, achieved when the channel is not in outage, results [7]

$$D_1 = 2^{-2R_{SD}}$$

The expected distortion is [5]

$$D = (1 - P_{out})D_1 + P_{out} \quad (13)$$

E. Discussion

The expected distortions achievable with the different strategies at a fixed $\bar{\gamma}$ of 10 dB as a function of outage probability are plotted in Figure 1. Figure 2 plots the coding rates at a fixed $\bar{\gamma}$ of 10 dB as a function of outage probability.

As expected, MD-OPT performs better at all probabilities, since it is designed to minimize expected distortion.

At low values for P_{out}, we can see that MD-OPT and MD-NJR have similar performance, MD-NMR and SD perform slightly worse, and DSD has the worst performance. This happens because the receiver is able to correctly decode both descriptions most of the time: since MD-NJR minimizes the joint description distortion (D_0), it delivers performance almost equivalent to the optimal coder. MD-NMR performs slightly worse because it is designed to minimize the individual description distortion (D_1), which leads to suboptimal values for D_0. DSD has worse performance than MD-NMR because it duplicates the same description of rate R_{DSD} and does not provide any gain when receiving both descriptions. Both DSD and MD-NMR have optimal individual descriptions, so the gap between them can be exclusively attributed to the capacity of MD-NMR to gain an advantage when receiving both descriptions. SD performs better than DSD because the channel outages are rare. In this case, using a single antenna that transmits at full power allowing higher rates brings lower distortions than duplicating a lower rate description over two reliable channels.

As P_{out} gets higher, SD and MD-NJR obtain worse performance and MD-NMR approaches MD-OPT. In this case the receiver decodes correctly only one description most of the time, so optimal performance is achieved with the MD-NMR coder, which minimizes D_1. DSD has slightly worse performance because, once again, it does not provide any gain when receiving both descriptions. Since this event now hap-

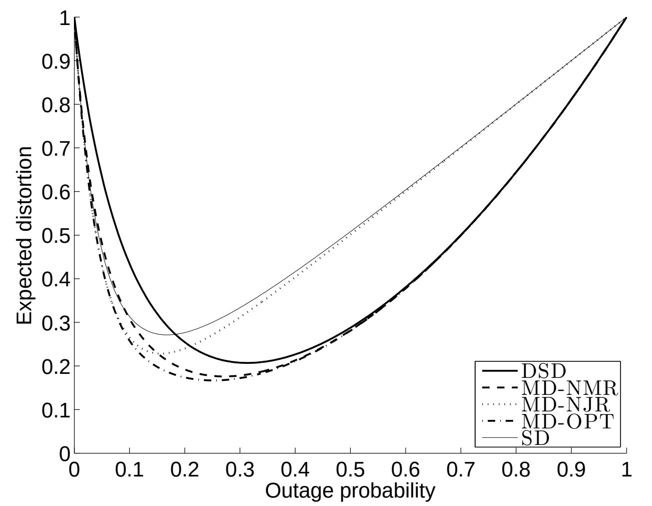

Fig. 1. Expected distortion vs. outage probability for the different strategies with $\bar{\gamma}$ = 10 dB.

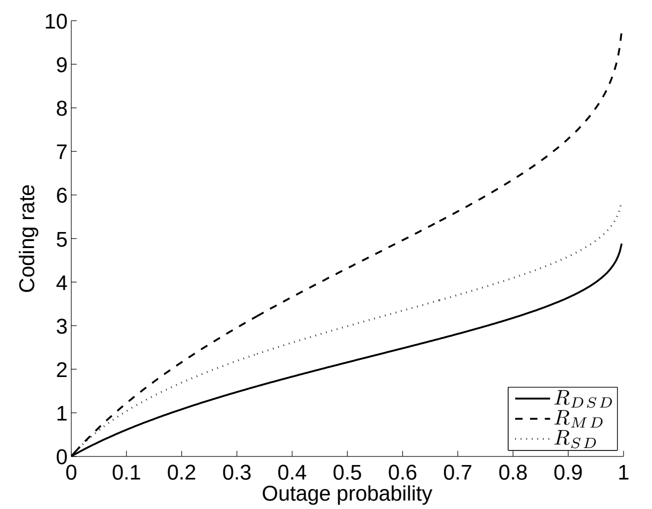

Fig. 2. Coding rates vs. outage probability with $\bar{\gamma}$ = 10 dB.

pens with lower probability, the gap between DSD and MD-NMR is reduced with respect to before. The poor performance of MD-NJR is simply due to the fact that this coder is designed to minimize D_0, while the even poorer performance of SD is due to the unreliability of the single channel.

At high values for P_{out}, the gap between DSD and MD-NMR gets very small and both strategies show optimal performance. This happens because the probability of receiving both descriptions is now very small.

MD-NJR performs significantly worse and has performance equivalent to SD. This last behavior can be explained by looking at Equation (7) and substituting into it the expressions of D_0 and D_1 for MD-NJR to get

$$D = (1 - P_{out})2^{-2R_{MD}} + P_{out} \tag{14}$$

which clearly shows that MD-NJR is equivalent to a SD scheme with rate R_{MD}. Since $R_{MD} > R_{SD}$, MD-NJR provides in general better performance than SD. When P_{out} gets sufficiently high, the second term on the RHS of Equations (13) and (14) dominates and both strategies show similar performance.

III. MIMO Strategies

We now consider a 2X2 MIMO system characterized by the channel matrix H, where each entry h_{ij} represents the channel gains between i-th receive antenna and j-th transmit antenna. The same assumptions of the previous section are valid also here so, in particular, the h_{ij} are i.i.d. zero mean complex Gaussian random variables.

We compare two different strategies for transmission of the same Gaussian source over this system. These strategies are Spatial Multiplexing (SM), which is used for transmitting a single description of the source, and Time Sharing (TS), which is used for transmitting a multiple description of the source.

A. Spatial Multiplexing

In the SM strategy, a SD code of rate R_{SM} from the SD coder is demultiplexed and coded into two independent half-rate substreams, which are sent over the two transmit antennas. At the receiver proper signal processing is done to recover the original full-rate stream.

The instantaneous capacity achievable with this strategy is given by [9]

$$C = \log_2 \det \left[I_2 + \frac{\bar{\gamma}}{2} H H^H \right] \tag{15}$$

where I_2 is the 2X2 identity matrix and H^H denotes the conjugate transpose of H. In a similar way as before, given a value for P_{out} the SD coder encodes the source using a rate R_{SM} such that $Pr\{C < R_{SM}\} = P_{out}$.

When the system is not in outage, the receiver observes a distortion D_1 equal to

$$D_1 = 2^{-2R_{SM}}$$

and the expected normalized distortion results

$$D = (1 - P_{out})D_1 + P_{out}$$

which is plotted in Figure 3.

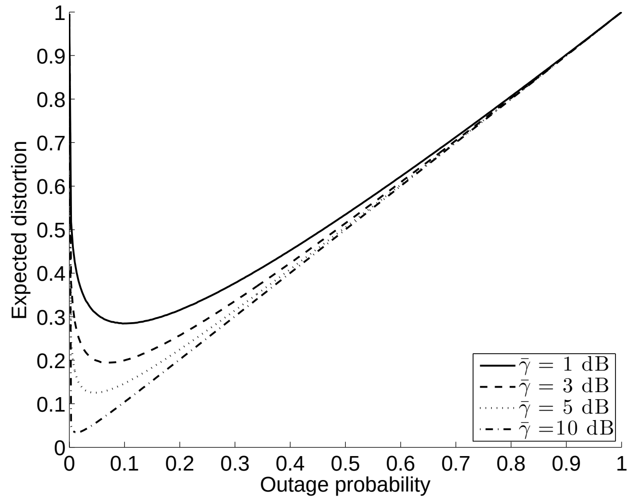

Fig. 3. Expected distortion vs. outage probability for SM with different values of $\bar{\gamma}$.

B. Time Sharing

With the TS strategy, two different symbols are transmitted over the two transmit antennas in two consecutive time slots. In each time slot only one antenna is transmitting, while the other is off. At the receiver a Maximal Ratio Combiner (MRC) [1] is used for combining the two signals received by the receive antennas in the same time slot. Thus, the TS technique yields two independent channels of gains $|h_{11}|^2 + |h_{21}|^2$ and $|h_{12}|^2 + |h_{22}|^2$ which can be used for MD or DSD.[1]

The instantaneous capacity C_j of j-th channel is thus [1]

$$C_j = \frac{1}{2} \log_2 \left(1 + \bar{\gamma} \sum_{i=1}^{2} |h_{ij}|^2 \right)$$

where the term $1/2$ arises because each channel is used only half the time.

Each description has the same rate $R_{TS}/2$ which is chosen, given P_{out}, such that

$$Pr\left\{ C_j < \frac{R_{TS}}{2} \right\} = P_{out}$$

As in the previous section, the MD-OPT coder chooses the values of D_0 and D_1 to minimize the expected distortion D, whose expression is given by Equation (12) in which R_{TS} is used instead of R_{MD}. The expected distortion is plotted in Figure 4.

C. Discussion

Figure 5 plots the expected distortions for SM and TS strategies at a fixed $\bar{\gamma}$ of 10 dB as a function of outage probability. Expected distortion for MD-OPT strategy is also plotted, for a comparison between MIMO strategies and parallel channel strategies.

As can be seen, at very low values for P_{out} the SM strategy achieves the best performance. As P_{out} gets higher,

[1]For brevity, here we consider only the optimal MD coder.

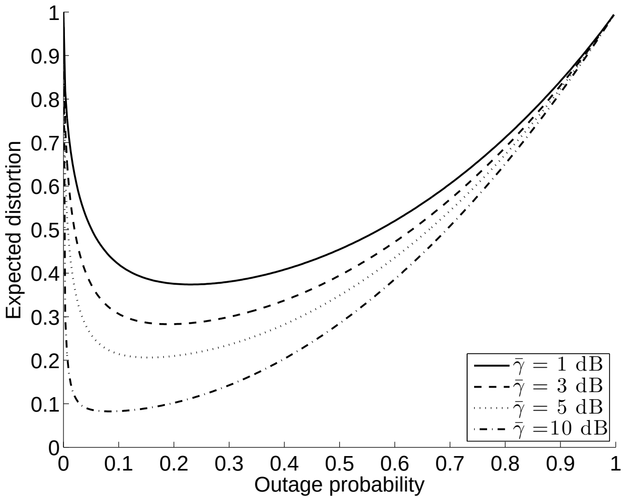

Fig. 4. Expected distortion vs. outage probability for TS with different values of $\bar{\gamma}$.

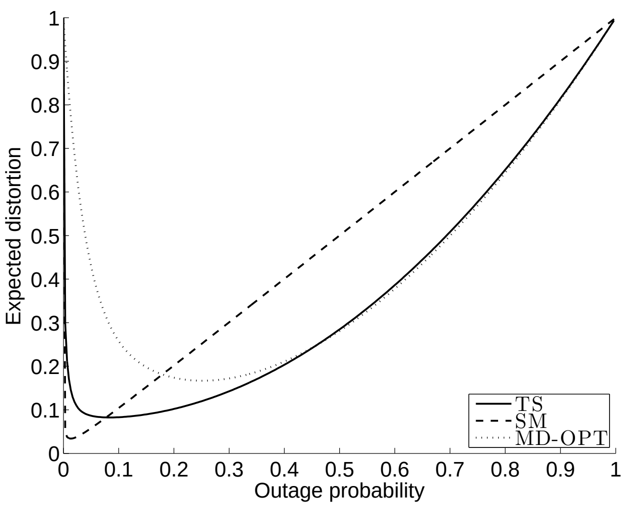

Fig. 5. Expected distortion vs. outage probability for various strategies with $\bar{\gamma}$ = 10 dB.

performance of SM rapidly worsens and TS obtains the best performance. As expected it turns out that, at least for useful values of P_{out}, both MIMO strategies outperform parallel channel strategy, suggesting that MIMO systems could be effectively used for achieving lower distortions.

It is interesting to observe that the SM strategy achieves its lowest distortions only for a very small range of values of P_{out}, while TS achieves its lowest distortions for a significantly higher range of values of P_{out}.

IV. Conclusions

We considered the lossy transmission of a Gaussian source over independent, parallel channels and a 2X2 MIMO channel. We evaluated mean squared error distortion at the receiver for duplicate single description coding and optimal, no excess joint rate and no excess marginal rate over a simplified system consisting of two parallel and independent channels. We compared these strategies to the traditional approach using single description coding over a single channel. We showed that using source coding diversity can significantly improve performance and that, if carefully chosen, multiple description coding can provide lower distortion with respect to duplicate single description coding.

We also compared two different strategies for transmission of the same Gaussian source over a standard 2X2 MIMO system. These strategies are Spatial Multiplexing (SM), which is used for transmitting a single description of the source, and Time Sharing (TS), which is used for transmitting a multiple description representation of the source. Results suggest that MIMO systems can provide noticeably lower distortions than parallel and independent channels.

References

[1] A. Goldsmith, *Wireless Communication*, Cambridge University Press, 2005.

[2] J. Balam and J. Gibson, "Path diversity and multiple descriptions with rate dependent packet losses", in *Proceedings of the Information Theory and Applications Workshop*, February 2006.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2006.

[4] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2003.

[5] S. Choudhury and J. Gibson, "Information Transmission Over Fading Channels", *IEEE Global Communications Conference*, November 2007

[6] A. Papoulis and S.U.Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. McGraw-Hill Science, 2001.

[7] T. Cover and J. Thomas. *Elements of Information Theory*, John Wiley & Sons, 1991.

[8] M. Effros, R. Koetter, A. Goldsmith, and M. Medard, "On source and channel codes for multiple inputs and outputs: Does multiple description beat space time?" in *Information Theory Workshop 2004*, Oct. 2004.

[9] G. J. Foschini and M. J. Gans, "On Limits of Wireless Communications in a Fading Environment", *Wireless Personal Communications*, Vol. 6, No. 3, pp. 311335, March 1998.