On Strategies for Source Information Transmission over MIMO Systems

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***Abstract*—We consider strategies for the lossy transmission of a zero mean Gaussian source over a 2×2 MIMO channel with Rayleigh fading. The source is represented either using a single description or a multiple description code, depending on each strategy characteristic. Performance is evaluated using normalized expected distortion at the receiver, as a function of outage probability. The first strategy employs repetition coding over the two transmit antennas for the transmission of a single description representation of the source. The second strategy uses a time-shared approach to the two transmit antennas, allowing for the transmission of a multiple description representation of the source. The third and fourth strategies are based on, respectively, the Alamouti scheme and spatial multiplexing, and both of these strategies are used for the transmission of a single description representation of the source. The results show that the spatial multiplexing strategy is able to achieve the lowest distortion, and also that it is possible, with the Alamouti strategy, to obtain similar performance at a lower complexity. We finally consider the outage rates of the different strategies and observe that if a system is designed to maximize the outage rate, the corresponding distortion observed at the receiver will not be minimized.**

I. Introduction

A Multiple-Input Multiple-Output (MIMO) system is a system that employs multiple antennas both at the transmitter and receiver. The first theoretical analyses of MIMO systems were developed by Winters [1], Foschini [2] and Telatar [3], as well as others, and since then there have been many research efforts on this subject. What mainly makes MIMO systems interesting is their potential ability to achieve an increase in system capacity or in link reliability without requiring additional transmission power or bandwidth [4].

In this work, we focus on the utilization of MIMO systems for the lossy transmission of source information. In particular, we want to compare several different strategies for the transmission of a zero mean Gaussian source over a 2×2 MIMO channel with Rayleigh fading. These strategies are based on techniques such as Repetition coding (REP) [5], Time Sharing (TS), the Alamouti scheme (ALM) [5], [6] and Spatial Multiplexing (SM) [5]. Depending on its characteristics, each strategy will be used either for the transmission of a Single Description (SD) or the transmission of a Multiple Description (MD) representation of this source.

In SD coding, a single stream of information describing the source is transmitted over a single channel. In MD coding [7], the source is represented using two different descriptions that are transmitted over two independent channels. If both descriptions are correctly received, they can be combined together at the receiver to obtain a reconstruction of the source at a certain quality. If only one of the two descriptions is correctly received, a reconstruction of the source is still possible but at a lower quality.

The comparison between the strategies is based on the normalized expected distortion at the receiver, evaluated as a function of outage probability. The results show that the SM strategy achieves the lowest distortion, but in general at the expense of high complexity. It is also shown that the ALM strategy, while being much lower in complexity, is able to achieve performance almost equivalent to SM. We finally consider the outage rates and show that, for each given strategy, the outage probability required for maximizing outage rate is different from the outage probability required for minimizing distortion at the receiver. This implies that the maximization of the outage rate might not be the optimal criterion for the design of a system, since in general it will not lead to the minimization of the distortion at the receiver.

II. Strategies for Information Transmission

A. System Model

The system considered in this work is the 2×2 MIMO system of Fig. 1. It is characterized by the channel matrix H, which has the form

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

Each entry h_{ij} of the channel matrix H represents the gain of the channel between j-th transmit antenna and i-th receive antenna. Each one of these channels is assumed to be independent, random and with very slow Rayleigh fading. The h_{ij} are then i.i.d. complex Gaussian random variables with zero mean and unit variance, which remain constant over the transmission of a large number of symbols.

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This research has been supported by the California Micro Program, Applied Signal Technology, Cisco, Dolby Labs, Inc., by Qualcomm, Inc, by NSF Grant Nos. CCF-0429884, CNS-0435527, CCF-0728646, and by the European Community under Seventh Framework Programme grant agreement ICT OPTIMIX n°INFSO-ICT-214625.

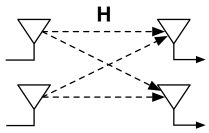

Fig. 1: 2×2 MIMO model.

Channel state information (CSI) is not available at the transmitter, which means that the transmitter does not have any information about the channel matrix H, except for its statistical distribution. Perfect CSI is assumed to be available at the receiver.

The total transmitted power by the transmit antennas is equal to P_t. If both transmit antennas are transmitting simultaneously, each antenna will transmit with equal power $P_t/2$, while, if only one antenna is transmitting at a given time, it can make use of full transmit power P_t. The noise at the receiver is i.i.d. AWGN noise, with the same average power N at each receive antenna. We denote with $\bar{\gamma}$ the ratio P_t/N and with γ_{ij} the instantaneous Signal to Noise Ratio (SNR) of the signal transmitted by the j-th antenna and received by the i-th antenna. Thus,

$$\gamma_{ij} = \frac{P_t}{N}|h_{ij}|^2 = \bar{\gamma}|h_{ij}|^2, \qquad i = 1, 2 \tag{1}$$

if only the j-th antenna is transmitting at a given time, and

$$\gamma_{ij} = \frac{P_t}{2N}|h_{ij}|^2 = \frac{\bar{\gamma}}{2}|h_{ij}|^2, \qquad i, j = 1, 2 \tag{2}$$

if both antennas are transmitting at the same time.

B. Repetition

The REP strategy is based on repetition coding [5]. The basic idea is to transmit the same symbol over the two transmit antennas in two consecutive time slots. In each time slot, only one of the two transmit antennas is used for transmission, while the other antenna is turned off.

Thus, in the first time slot the symbol S_1 is transmitted on the first transmit antenna and it is observed by the receiver through the two channels with gains h_{11} and h_{21}. In the second time slot, the same symbol S_1 is transmitted on the second transmit antenna and it is observed by the receiver through the two channels with gains h_{12} and h_{22}. A Maximal Ratio Combiner (MRC) [4] is then used at the receiver to optimally combine the four signals received by the two receive antennas in the two different time slots.

The instantaneous SNR γ of the signal at the output of the MRC is given by the sum of the instantaneous SNRs γ_{ij} of its input signals [4], that are given by Eq. (1)

$$\gamma = \sum_{i,j=1}^{2} \gamma_{ij} = \bar{\gamma} \sum_{i,j=1}^{2} |h_{ij}|^2$$

In this way, a single channel is obtained from the four independent channels available in our MIMO system. This strategy is then suitable for the transmission of a SD representation of the source.

The instantaneous capacity of this single channel is given by [4]

$$C = \frac{1}{2}\log_2\Big(1+\gamma\Big) = \frac{1}{2}\log_2\left(1+\bar{\gamma}\sum_{i,j=1}^{2}|h_{ij}|^2\right)$$

where the factor $1/2$ arises because we are transmitting the same symbol over two consecutive time slots.

The source coding rate R_{REP} of the SD coder is chosen to be equal to the outage capacity at a given value for the outage probability P_{out}, i.e. it is chosen such that

$$Pr\Big\{C < R_{REP}\Big\} = P_{out}$$

Thus, with probability $1 - P_{out}$ the system is not in outage, which means that it can support the transmission at a rate R_{REP} with an arbitrarily small probability of error, since its capacity is higher than R_{REP} [8]. In such case, the receiver is able to reconstruct the source information with a distortion D_1 equal to [8]

$$D_1 = 2^{-2R_{REP}}$$

If the system results in outage, which happens with probability P_{out}, the receiver is not able to correctly decode the transmitted information with an arbitrarily small probability of error and achieves a distortion equal to 1.

The expected distortion D at the receiver is then

$$D = (1 - P_{out})D_1 + P_{out}$$

The outage rate R_{REP}^{out}, defined as the average rate correctly received over many transmission bursts [4], is given by

$$R_{REP}^{out} = (1 - P_{out})R_{REP}$$

C. Time Sharing - Multiple Description (TS-MD)

In this strategy a TS approach is adopted to obtain two independent channels from the MIMO system. The idea behind this strategy is to transmit two different symbols over the two transmit antennas in two consecutive time slots. In each time slot, only one of the two transmit antennas is used for transmission, while the other antenna is turned off. Thus, in the first time slot the first symbol S_1 is transmitted over the first antenna and it is observed by the receiver through the two channels with gains h_{11} and h_{21}. In the second time slot, the second symbol S_2 is transmitted over the second antenna and it is observed by the receiver through the two channels with gains h_{12} and h_{22}. The receiver will then combine the two signals received in the same time slot using a MRC.

Since each received signal has a SNR given by Eq. (1), the signal at the output of the MRC in the j-th time slot has a SNR equal to [4]

$$\gamma_j = \sum_{i=1}^{2} \gamma_{ij} = \bar{\gamma} \sum_{i=1}^{2} |h_{ij}|^2$$

In this way, two independent channels are effectively created in the two time slots, making this strategy suitable for the transmission of a MD representation of the source.

The channel at the j-th time slot has an instantaneous capacity C_j equal to [4]

$$C_j = \frac{1}{2}\log_2\left(1+\gamma_j\right) = \frac{1}{2}\log_2\left(1+\bar{\gamma}\sum_{i=1}^{2}|h_{ij}|^2\right) \quad (3)$$

where the factor $1/2$ arises because each channel is used only half of the time.

The side description rate $R_{MD}/2$, which equals the transmitted rate over each channel, is chosen to be equal to the outage capacity for a given P_{out}, i.e. is chosen such that

$$Pr\left\{C_j < \frac{R_{MD}}{2}\right\} = P_{out}$$

The expected distortion D at the receiver is then given by

$$D = (1-P_{out})^2 D_0 + 2P_{out}(1-P_{out})D_1 + P_{out}^2 \quad (4)$$

where D_0 and D_1 are the distortions achieved by the receiver when observing, respectively, both descriptions or only one of the two descriptions.

Depending on the type of MD coder used, D_0 and D_1 can have different expressions [9] and different TS-MD strategies can be obtained. The No Excess Marginal Rate coder (MD-NMR) [9] is employed in the TS-MD-NMR strategy. The side descriptions are then rate distortion optimal and the distortions have the following expressions [9], [10]

$$\begin{aligned} D_0 &= \frac{2^{-R_{MD}}}{2-2^{-R_{MD}}} \\ D_1 &= 2^{-R_{MD}} \end{aligned}$$

The No Excess Joint Rate coder (MD-NJR) [9] is employed in the TS-MD-NJR strategy. Here the joint description is rate distortion optimal and the distortions have the following expressions [9], [10]

$$\begin{aligned} D_0 &= 2^{-2R_{MD}} \\ D_1 &= \frac{1}{2}\left(1+2^{-2R_{MD}}\right) \end{aligned}$$

The optimal coder (MD-OPT) [10] is employed in the TS-MD-OPT strategy. In this case, neither the side descriptions nor the joint description is rate distortion optimal, but they are chosen to minimize the expected distortion D in Eq. (4) for a given P_{out}. The distortions D_0 and D_1 are given by the following expression [9], [10]

$$\left(D_0, D_1\right) = \left(a, \frac{1+a}{2} - \frac{1-a}{2}\sqrt{1-\frac{2^{-2R_{MD}}}{a}}\right)$$

with

$$a \in \left[2^{-2R_{MD}}, \frac{2^{-R_{MD}}}{2-2^{-R_{MD}}}\right]$$

Thus, the MD-OPT coder chooses the proper value for a to minimize the expected distortion D.

The outage rate R_{MD}^{out} is given by

$$\begin{aligned} R_{MD}^{out} &= (1-P_{out})^2 R_{MD} + 2P_{out}(1-P_{out})\frac{R_{MD}}{2} \\ &= (1-P_{out})R_{MD} \end{aligned}$$

D. Alamouti

This strategy employs the Alamouti scheme [6], [5] to obtain two independent channels from the MIMO system. Since both channels have the same gain given by $\sum_{i,j=1}^{2}|h_{ij}|^2$ [5], it is evident that it is impossible to have, for a given realization of the channel matrix H, one channel in outage and the other not in outage, i.e. both channels can only be simultaneously in outage or simultaneously not in outage.[1] This strategy is then not suitable for the transmission of a multiple description representation of the source, as also pointed out in [10]. Instead, it could be used for the transmission of a single description representation, demultiplexing it into two half-rate substreams which are then transmitted over the two channels.

The signals at the output of the Alamouti decoder have the same instantaneous SNR γ, equal to the sum of the SNRs of the signals on each branch [4]. Thus, from Eq. (2) we have

$$\gamma = \sum_{i,j=1}^{2}\gamma_{ij} = \frac{\bar{\gamma}}{2}\sum_{i,j=1}^{2}|h_{ij}|^2$$

The instantaneous capacity of this system is then given by [11]

$$C = \log_2\left(1+\gamma\right) = \log_2\left(1+\frac{\bar{\gamma}}{2}\sum_{i,j=1}^{2}|h_{ij}|^2\right)$$

and the source coding rate R_{ALM} is chosen such that

$$Pr\left\{C < R_{ALM}\right\} = P_{out}$$

The expected distortion is then

$$D = (1-P_{out})D_1 + P_{out}$$

where D_1 is the distortion achieved by the receiver when the system is not in outage, which is equal to [8]

$$D_1 = 2^{-2R_{ALM}} \quad (5)$$

The outage rate R_{ALM}^{out} is given by

$$R_{ALM}^{out} = (1-P_{out})R_{ALM}$$

E. Spatial Multiplexing

In the SM strategy [5], a single symbol stream is first demultiplexed and encoded into two separate and independent substreams. Each substream is then transmitted simultaneously over each transmit antenna and, at the receiver, an optimal joint decoder is employed for retrieving the original symbol stream.

Since this strategy requires one single symbol stream, it can only be used for the transmission of a SD representation of the source.

The instantaneous capacity achievable with this strategy is given by [2]

$$C = \log_2\det\left(I_2 + \frac{\bar{\gamma}}{2}HH^H\right) \quad (6)$$

[1] This is true also because the transmitted rate on each channel is the same, which is the only case of interest for us.

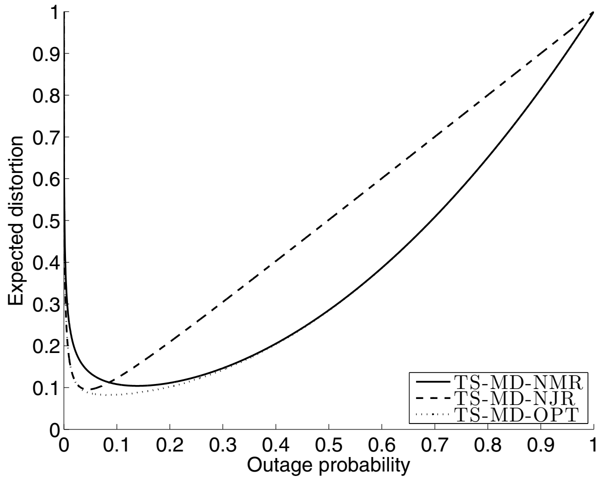

Fig. 2: Expected distortion vs. outage probability for the different TS strategies with $\bar{\gamma}$ = 10 dB.

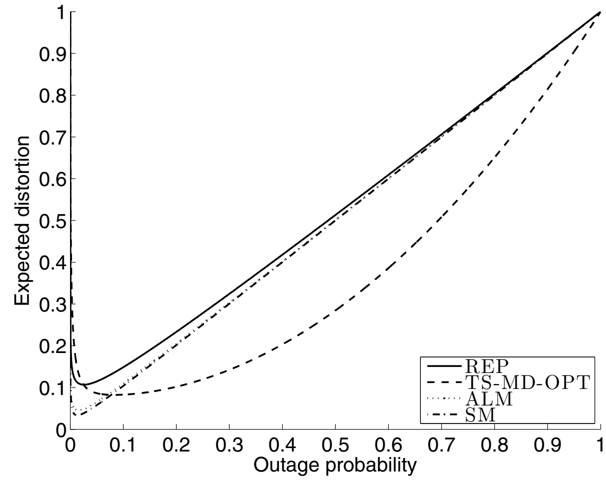

Fig. 3: Expected distortion vs. outage probability for the different strategies with $\bar{\gamma}$ = 10 dB.

where I_2 is the 2×2 identity matrix and H^H denotes the conjugate transpose of the channel matrix H. In a similar way as before, given the outage probability the source coding rate R_{SM} is chosen such that

$$Pr\{C < R_{SM}\} = P_{out}$$

The expected distortion D at the receiver is then

$$D = (1 - P_{out})D_1 + P_{out}$$

where

$$D_1 = 2^{-2R_{SM}} \quad (7)$$

is, as usual, the distortion achieved when the system is not in outage.

The outage rate R_{SM}^{out} is given by

$$R_{SM}^{out} = (1 - P_{out})R_{SM}$$

III. Discussion

We begin the discussion by comparing only the three TS-MD strategies. Then, we compare TS-MD-OPT with the remaining three strategies.

Figure 2 compares the expected distortions achievable with the TS strategies at a fixed $\bar{\gamma}$ of 10 dB. These results can be explained using the same observations we made in [12], where we considered MD strategies over two parallel and independent fading channels. For completeness, we now briefly restate here these conclusions.

As expected, TS-MD-OPT achieves the lowest distortions, since it is designed to minimize Eq. (4). At low outage probabilities, both descriptions are correctly decoded most of the time and optimal performance is achievable with the TS-MD-NJR strategy, since it is designed to minimize the distortion D_0. As the outage probability gets higher, the receiver becomes able to correctly decode only one description most of the time and TS-MD-NMR achieves optimal performance, since it is designed to minimize the distortion D_1.

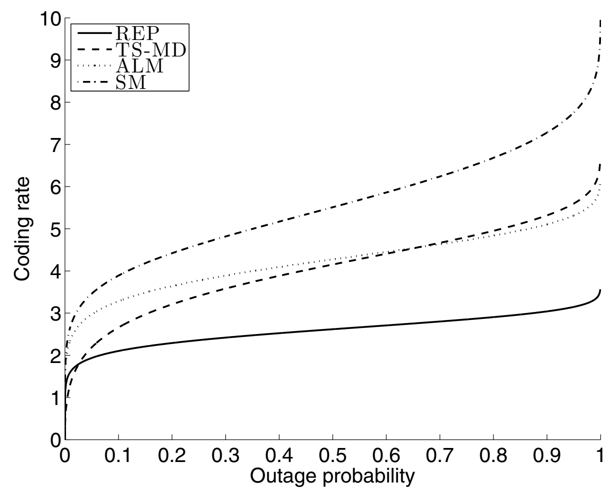

Fig. 4: Source coding rates vs. outage probability for the different strategies with $\bar{\gamma}$ = 10 dB.

Figure 3 compares the remaining strategies and TS-MD-OPT at a fixed $\bar{\gamma}$ of 10 dB. As can be seen, the lowest distortions are achieved with the SM strategy. However, this performance comes at the expense of complexity, mainly due to the presence of the joint decoder at the receiver. Interestingly, the ALM strategy, which can be employed for reducing this complexity, shows only a very small loss in performance with respect to SM. Looking at the source coding rates of the different strategies, reported in Fig. 4, it can be seen that ALM obtains rather high coding rates, but still significantly lower than those of SM. This observation, at a first analysis, might erroneously lead to the expectation of a more evident difference in performance between these two strategies. In fact, it must be recalled that the distortions D_1 are exponential decaying functions of the coding rate (see Eqs. (5) and (7)). So, due to this type of dependency, the

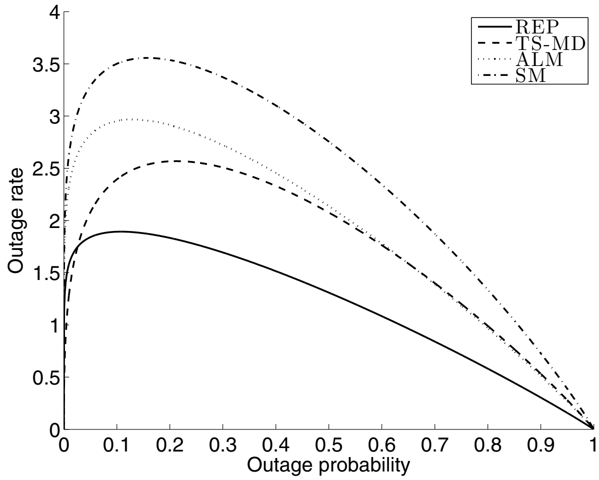

Fig. 5: Outage rate vs. outage probability for the different strategies with $\bar{\gamma}$ = 10 dB.

distortions D_1 are very similar even though the coding rates are significantly different.

Returning to Fig. 3, as the outage probability grows performance of SM and ALM quickly worsen and lowest distortions become achievable with the TS-MD-OPT strategy. This happens because SM and ALM are both transmitting over a single unreliable channel, while TS-MD-OPT employs path diversity over two independent and equally unreliable channels reducing the overall system outage probability. The REP strategy has in general the worst performance, except for very low values of outage probabilities where it performs slightly better than TS-MD-OPT.

We now consider the outage rates achievable with the various strategies, plotted in Fig. 5. In [13] it has been shown that, when considering the lossy transmission of information over a single channel with very slow Rayleigh fading, designing the system to maximize outage rate does not lead to the minimization of the distortion at the receiver. Inspired by this observation, we want to determine if the same result applies to our MIMO case.

We denote by p_d the outage probability that minimizes expected distortion and by p_r the outage probability that maximizes outage rate. Table I shows the values of p_d, p_r, the corresponding distortions and the percent differences in distortion for the various strategies with $\bar{\gamma}$ = 10 dB, obtained from Figs. 3 and 5. As can be seen, the outage probabilities that minimize distortion are very different from the outage probabilities that maximize outage rate. Thus, if the system is designed to maximize outage rate instead of minimizing distortion, suboptimal performance is achieved. In some cases this approach could result in much higher distortions, as in the SM strategy where the percent difference between the two distortions is almost 370%. TS-MD-OPT is the less sensitive strategy to this design error, but still has a distortion that is more than 28% higher than the minimum distortion.

TABLE I

p_d, p_r AND RESPECTIVE DISTORTIONS FOR THE VARIOUS STRATEGIES WITH $\bar{\gamma}$ = 10 dB.

	p_d	p_r	$D(p_d)$	$D(p_r)$	$\Delta D\%$
ALM	0.015	0.130	0.0458	0.1372	199.41
TS-MD-OPT	0.084	0.212	0.0825	0.1058	28.24
REP	0.026	0.108	0.1069	0.1551	45.01
SM	0.013	0.157	0.0338	0.1589	369.76

IV. Conclusions

We considered strategies for the lossy transmission of a zero mean Gaussian source over a 2×2 MIMO channel with Rayleigh fading. These strategies are based on Repetition coding, the Alamouti scheme and Spatial Multiplexing for the transmission of a single description representation of the source, and on Time Sharing for the transmission of a multiple description representation. We showed that the lowest distortions are achievable with SM strategy, but at a high complexity. We also showed that the Alamouti strategy has performance almost equivalent to SM, but at a lower complexity. We finally considered the outage rates of the different strategies and observed that in a system designed to maximize the outage rate, the corresponding distortion observed at the receiver is not minimized.

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