Information Transmission Over Fading Channels

Sayantan Choudhury and Jerry D. Gibson
Department of Electrical and Computer Engineering
University of California, Santa Barbara
Email: {sayantan, gibson}@ece.ucsb.edu

***Abstract*— We consider the lossy transmission of source information over Rayleigh fading channels. We investigate the utility of two channel capacity definitions, ergodic capacity, where it is assumed that the channel cycles through all fading states, and outage capacity, where the source is transmitted at a constant rate with a specified outage probability. We also study the outage rate and expected source distortion for different outage probabilities. It is observed that the outage probabilities required to maximize outage rate and minimize expected distortion are quite different. This implies that schemes based on maximizing capacity might not lead to the most efficient design for lossy transmission of source information over wireless networks. We show that minimizing expected distortion over a wireless link does not necessarily minimize the variance of the distortion, and hence parameter selection based on minimizing expected distortion can lead to a high distortion for a specific realization. We also introduce different performance measures that take into account the variance of source distortion and might be more suitable for source transmission over fading channels. Finally, we observe that in a Rayleigh fading channel in the presence of channel state information (CSI) at both the transmitter and receiver, in addition to a capacity distribution, there is a source distortion distribution at the receiver for a memoryless Gaussian source. A careful investigation of the capacity and source distortion distributions reveal that the probability of achieving the average source distortion increases with an increase in average signal to noise ratio (SNR) while the probability of achieving the average capacity does not change significantly with SNR.**

I. Introduction

There is considerable interest in the transmission of source information such as voice and video over wireless networks. Several cross-layer design schemes have been proposed that improve the physical, link and network layers using a joint optimization framework [1]. Many of these cross-layer schemes aim at maximizing the effective throughput by varying the transmission rate, number of retransmissions, payload size, etc. [2]. In this work, we contrast the capacity maximization problem with that of a distortion minimization problem. We observe that the operating regions for capacity maximization are quite different from those that minimize distortion. Moreover, we also highlight that minimizing the expected distortion in a fading environment might not be the best performance indicator due to the large variation in the individual realizations.

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There has also been significant theoretical interest in evaluating source fidelity over fading channels [3]. In [3], the transmission of a continuous source over a two-hop fading channel is considered. It was observed that a smart relay for a two-hop channel that optimizes the transmission rate over the second link can at most give a performance gain of 3 dB over a relay that does not alter the rate over the individual links. In [4], the transmission of a continuous amplitude source over a quasi-static fading channel is considered. Three different source and channel coding strategies are compared with the aim of minimizing the distortion exponent, which indicates the decay of expected distortion with increasing SNR [4]. In [5], *source coding diversity* is contrasted with *channel coding diversity* for minimizing the end-to-end distortion as a function of SNR. It is shown that both schemes have certain advantages and the choice of preferring one over the other should be based not only on obtaining a large distortion exponent but also by considering system implementation, cost benefits, etc [5].

Motivated by our interest in source transmission over fading channels, in this paper, we compare the source distortion for two definitions of channel capacity, ergodic capacity and outage capacity, with and without channel state information (CSI) at the transmitter [6]. Perfect CSI at the receiver is assumed for both cases. Ergodic capacity is calculated based on the assumption that channel fading transitions through all possible fading states, and thus this definition might not be very useful in practice for source transmission with fixed delay constraints. Schemes that achieve a certain outage capacity transmit data at the maximum allowable rate for a specified outage probability. We show that the source distortion at the receiver is quite different for source transmission schemes that achieve ergodic capacity compared to schemes that achieve a certain outage rate. Moreover, schemes based on joint source/channel coding aim to minimize the expected distortion over all the fading realizations. However, we show that in a fading channel, due to the large variance in individual realizations, there are other peformance indicators that might be more useful for source transmission.

We also evaluate the distribution of achieved source distortion for an $i.i.d$ Gaussian source transmitted over a Rayleigh fading link with CSI available at both the transmitter and receiver. In such a scenario, the transmitter adjusts the source rate to achieve the instantaneous channel capacity and hence, there is no outage. We evaluate the capacity and distortion distribution as a function of average SNR and observe that the average distortion cannot be guaranteed with high reliability at low SNRs.

The paper is outlined as follows. In the next section, we provide a brief description of two different notions of channel capacity used in wireless communications and evaluate the expected source distortion. In Section III, we compare the expected distortion at the receiver for two schemes of source transmission, namely, maximizing the outage rate and minimizing the expected distortion. We also compare the expected distortion and the variance of the distortion for a single-hop channel. In Section IV, we study the distribution of capacity and source distortion for different SNRs and observe that the expected distortion cannot be achieved with a high reliability for low SNRs. Section V states some conclusions.

II. Ergodic capacity, outage capacity and expected distortion

We discuss the classical definitions of channel capacity for fading channels and their applications to source transmission. The system model consists of a single hop channel as shown in Fig. 1. We specifically examine the case where CSI is unavailable at the transmitter but available at the receiver. A detailed description of the capacity of flat fading channels is available in [6].

A. System Model

As shown in Fig. 1, we consider a discrete time channel with stationary and ergodic time varying gain 'a' and additive white Gaussian noise (AWGN) 'n'. A block fading channel gain is assumed that remains constant over a blocklength and changes for different blocks based on a Rayleigh distribution. At the receiver, the instantaneous signal to noise ratio (SNR) γ is then given by an exponential distribution:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp(-\frac{\gamma}{\bar{\gamma}}), \qquad \gamma \geq 0 \tag{1}$$

where $\bar{\gamma}$ is the average SNR.

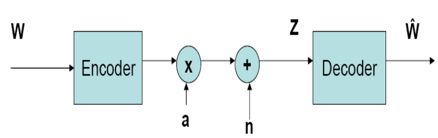


Fig. 1. System model

B. Channel Side information at Receiver

We consider the case where CSI is known at the receiver, i.e., γ is known at the receiver for every time instant. In practice, this is accomplished using channel estimation techniques [7]. Moreover, the distribution of γ is known at both the transmitter and receiver. Traditionally, for capacity analyses, two channel capacity definitions are used, namely *ergodic capacity* and *outage capacity*.

1) Shannon (Ergodic) capacity: When CSI is not available at the transmitter, the source data is transmitted at a constant rate, and data transmission takes place over all fading states, including deep fades where the data is lost, and hence the effective capacity is significantly reduced. The Shannon capacity of a fading channel with receiver CSI only for an average power constraint $\bar{P}$ is given by [6]

$$C_{erg} = \int_0^\infty B \log_2(1+\gamma)p(\gamma)d\gamma \tag{2}$$

where B is the received signal bandwidth. This is also referred to as ergodic capacity since it is the average of the instantaneous capacity for an AWGN channel with SNR γ given by $B\log_2(1+\gamma)$.

For an $i.i.d$ Gaussian source sequence with mean zero and variance σ^2, the distortion rate function with squared error distortion is given by [8]

$$D(R) = \sigma^2 2^{-2R} \tag{3}$$

The distortion at the receiver for the constant rate transmission scheme is then given by

$$D(C_{erg}) = \sigma^2 2^{-2C_{erg}} \tag{4}$$

However, this notion of ergodic capacity might not be a suitable performance metric for evaluating the distortion of sources with delay constraints. As pointed out in [9], a very long Gaussian codebook is required for achievability of Shannon capacity, the length being dependent on the dynamics of the fading process. In particular, it must be long enough for the fading to reflect its ergodic nature, i.e. the symbol time T must be much larger than the coherence time T_{coh}, defined to be the time over which the channel is significantly correlated.

2) Outage capacity: Outage capacity is used for slowly varying channels where the instantaneous SNR γ is assumed to be constant for a large number of symbols. Unlike ergodic capacity, schemes designed to achieve outage capacity allow for channel errors. Hence, in deep fades these schemes allow the data to be lost and a higher data rate can be maintained compared to schemes achieving Shannon capacity, where the data needs to be correctly received over all fading states [6].

Specifically, a design parameter P_{out} is selected that indicates the probability that the system can be in outage. Corresponding to this outage probability, there is a minimum received SNR, γ_{min}, given by $P_{out} = p(\gamma < \gamma_{min})$. For received SNRs below γ_{min}, the received symbols cannot be successfully decoded with probability 1, and the system declares an outage. Since the instantaneous CSI is not known at the transmitter, this scheme transmits using a constant data rate $C_{out} = B\log_2(1+\gamma_{min})$ which is successfully decoded with probability $1-P_{out}$. Hence the average outage rate R_{out} correctly received over many transmission bursts is given by

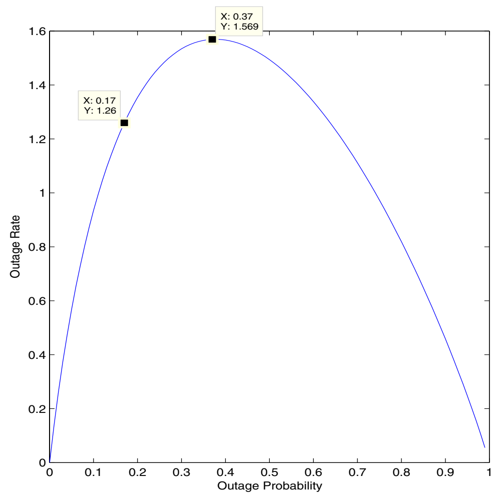
$$R_{out} = (1-P_{out})B\log_2(1+\gamma_{min}) \tag{5}$$

The expected distortion at the receiver for an $i.i.d$ $N(0,\sigma^2)$

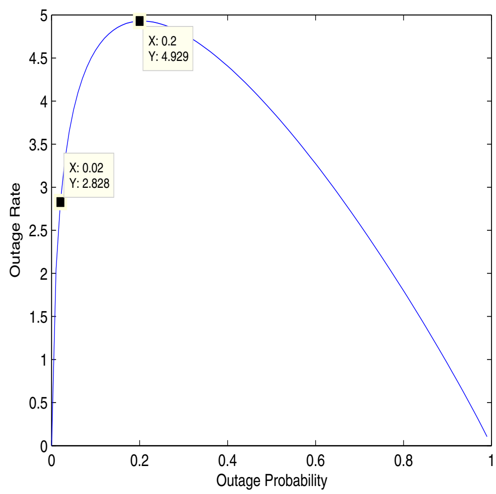
source for the above scheme is given by [3]

$$E[D] = D(B\log_2(1+\gamma_{min}))(1-P_{out}) + \sigma^2 P_{out} \qquad (6)$$

This can be interpreted as follows: either the source data is correctly decoded, resulting in a received distortion $D(B\log_2(1+\gamma_{min}))$, or there is an outage in which case the received variance is the source variance σ^2.



(a) Outage rate as a function of outage probability for an average SNR of 10 dB

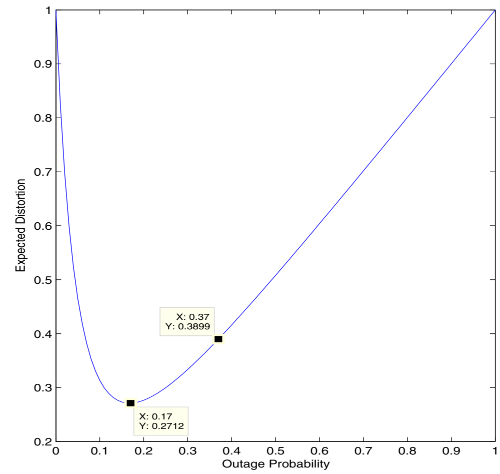


(b) Outage rate as a function of outage probability for an average SNR of 25 dB

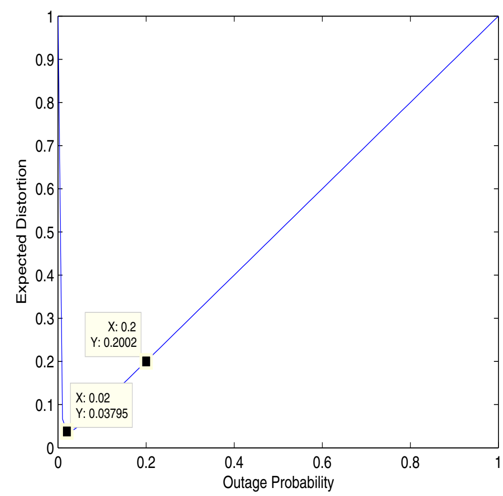
Fig. 2. Outage rate for various outage probabilities

III. Information transmission over outage channels

We compare the two approaches of maximizing outage rate and minimizing expected distortion for information transmission and examine the variance of the distortion at the receiver.



(a) Expected distortion as a function of outage probability for an average SNR of 10 dB



(b) Expected distortion as a function of outage probability for an average SNR of 25 dB

Fig. 3. Expected distortion for various outage probabilities

A. *Maximizing outage rate and minimizing expected distortion*

We revisit outage capacity in the context of information transmission. A Rayleigh fading channel as in Fig. 1 is considered and the bandwidth B and variance σ^2 are normalized to unity. P_{out} is given by [10]

$$P_{out} = p(\gamma < \gamma_{min}) = 1 - \exp(-\frac{\gamma_{min}}{\bar{\gamma}}) \qquad (7)$$

Equivalently, from Eq. (7), we obtain

$$\gamma_{min} = -\bar{\gamma}\log(1-P_{out}) \qquad (8)$$

P_{out} is a design parameter so the average rate correctly received (R_{out}) in Eq. (5) can be maximized as a function of P_{out}. Similarly, the expected distortion in Eq. (6) can be minimized as function of P_{out}. Perhaps surprisingly, it turns out that the outage probabilities in the two scenarios are quite different.

Figure 2 plots the outage rate as a function of outage probability for average SNRs of 10 dB and 25 dB. In Fig. 3, we plot the expected distortion as a function of outage probability for the same average SNRs of 10 dB and 25 dB.

There are optimal outage probabilities, p_c and p_d, that maximize outage rate and minimize expected distortion, respectively. From Fig. 2(a), for an average SNR of 10 dB, the outage probability of 0.37 maximizes outage rate whereas as evident from Fig. 3(a), an outage probability of 0.17 minimizes expected distortion. Lower outage probabilities are required for minimizing source distortion since source distortion is an exponentially decaying function of data rate and the lower source distortion obtained by employing higher data rates is offset by the fact that most of the transmissions are not successfully received. Figures 2(b) and 3(b) plot the outage rate and expected distortion, respectively, for various outage probabilities at an average SNR of 25 dB. The outage probability to minimize expected distortion is lower and the outage probability region to achieve minimum expected distortion becomes narrower.

B. Expected distortion and variance of distortion

We evaluate the standard deviation of source distortion as a function of outage probability. The expected value of distortion squared is given by:

$$E[D^2] = D^2(R_s)(1 - P_{out}) + \sigma^4 P_{out} \tag{9}$$

Therefore, the variance of distortion is given by

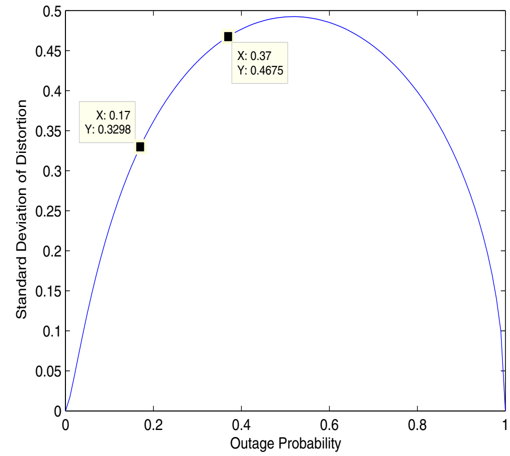
$$\begin{aligned} Var(D) &= E[D^2] - E[D]^2 \\ &= D^2(R_s)(1 - P_{out}) + \sigma^4 P_{out} \\ &\quad - [D(R_s)(1 - P_{out}) + \sigma^2 P_{out}]^2 \\ &= P_{out}(1 - P_{out})[\sigma^2 - D(R_s)]^2 \end{aligned} \tag{10}$$

and the standard deviation is

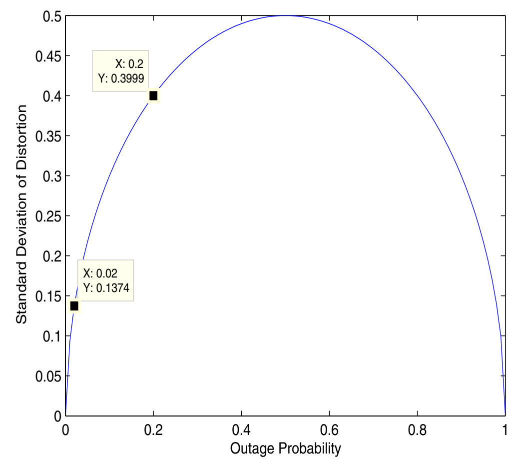
$$\sigma_{distn} = \sqrt{P_{out}(1 - P_{out})}\, (\sigma^2 - D(B \log_2(1+\gamma_{min}))) \tag{11}$$

In Fig. 4, we plot the standard deviation of the received distortion for average SNRs of 10 dB and 25 dB. We see that the scheme that maximizes outage rate from Fig. 2 has a significantly higher standard deviation of distortion compared to the scheme that minimizes expected distortion from Fig. 3. However, even the approach of minimizing expected distortion can have a high variance. Thus, a particular channel realization might have a very high distortion though, on the average, the expected source distortion is minimized. Hence for source transmission, especially with a quality constraint, there could be other performance criteria which are more beneficial for end-to-end performance. For instance, one formulation would be to minimize the expected distortion,

$$min_{P_{out}} E[D] = D(B \log_2(1 + \gamma_{min}))(1 - P_{out}) + \sigma^2 P_{out} \tag{12}$$



(a) Standard deviation of source distortion as a function of outage probability for an average SNR of 10 dB



(b) Standard deviation of source distortion as a function of outage probability for an average SNR of 25 dB

Fig. 4. Standard deviation of source distortion for various outage probabilities

subject to the constraint,

$$\sigma_{distn} = \sqrt{P_{out}(1 - P_{out})}\, (\sigma^2 - D(B \log_2(1+\gamma_{min}))) \leq \sigma_0 \tag{13}$$

where σ_0 is the maximum allowable standard deviation of distortion.

Another alternative would be a weighted minimization of the expected distortion and standard deviation of distortion where the weights are chosen according to the application requirements,

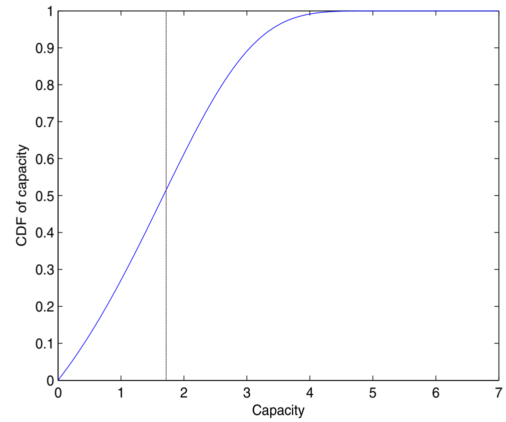
$$min_{P_{out}} w_1 E[D] + w_2 \sigma_{distn} \tag{14}$$

where w_1 and w_2 are the weights for expected distortion and standard deviation of distortion, respectively. Such an approach for video transmission is explored in [11].

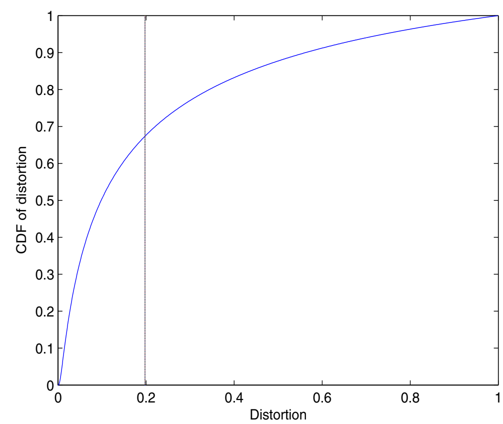
Yet another technique would be to choose the operating regions where a certain probability (that a specified distortion value is exceeded) can be achieved. The choice of objective function to minimize depends on the application, computation complexity, ease of implementation, etc. In [12], [13], we propose a performance indicator for speech transmission over wireless networks, MOS_x, that guarantees a low distortion for a large percentage of realizations. In [14], we propose a statistical video quality indicator $PSNR_{r,f}$ as PSNR achieved by f% of the frames in each one of the r% of the realizations. Using a subjective experiment, we show that $PSNR_{r,f}$ correlates significantly better than average PSNR to the perceptual quality [14].

The standard deviation of distortion for an average SNR of 25 dB is shown in Fig. 4(b). For the outage rate maximization scheme, the standard deviation of distortion at an SNR of 10 dB is 0.4675 whereas at an average SNR of 25 dB, it reduces to 0.3999. However, quite interestingly, the difference in the standard deviations of distortions between the two schemes increases with SNR from 0.1377 at an average SNR of 10 dB to 0.2625 at an average SNR of 25 dB.

It can also be seen from Fig. 4(a), that if a constraint is imposed on the variance, for instance, a maximum standard deviation of 0.1, the outage probability has to be less than appromixately 0.05. The minimum expected distortion for outage probabilities less than 0.1 for an average SNR of 10 dB is nearly 0.3 from Fig. 3(a), which is higher than the unconstrained minimum expected distortion of 0.27. Thus for constrained variance schemes, either the operating SNR has to be increased or the system has to operate at an expected distortion larger than the unconstrained minimum expected distortion.



(a) Distribution of capacity



(b) Distribution of distortion

Fig. 5. CDF of capacity and distortion for an average SNR of 5 dB

IV. SOURCE DISTORTION WITH PERFECT CSI AT BOTH TRANSMITTER AND RECEIVER

When CSI is available only at the receiver and not at the transmitter, the source rate cannot be adapted based on channel conditions. However, in the event that CSI is available at the transmitter as well, the source rate can be adapted based on channel conditions, and this in turn induces a distribution of source distortion at the receiver as discussed in the following. There is no notion of outage in this case since the transmitter can always adjust the source rate to achieve the instantaneous channel capacity [6].

The Shannon capacity when CSI is available at the transmitter but without power adaptation is the same as in Eq. (2), i.e. the transmitter side information does not increase the channel capacity unless the power is also adapted [6]. However, since the CSI is available at the transmitter, the source rate can be adapted instantaneously and this leads to a more practical system design. For instance, it is not required to have very long codes that average over all fading states. It is quite interesting to investigate the probability distribution of distortion induced at the receiver by this variable rate scheme and to determine, in particular, the probability of achieving the expected distortion.

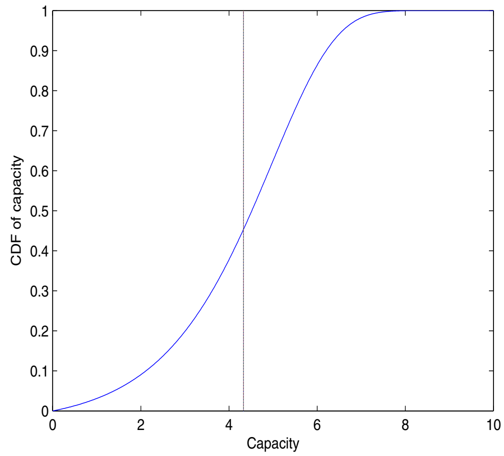
The cumulative distribution function (CDF) of capacity can be derived as

$$\begin{aligned} F_c(c) &= Pr\{C(\gamma) \le c\} = Pr\{B \log_2(1+\gamma) \le c\} \\ &= Pr\{\gamma \le 2^{\frac{c}{B}} - 1\} \\ &= 1 - \exp(\frac{1 - 2^{\frac{c}{B}}}{\bar{\gamma}}) \end{aligned} \quad (15)$$

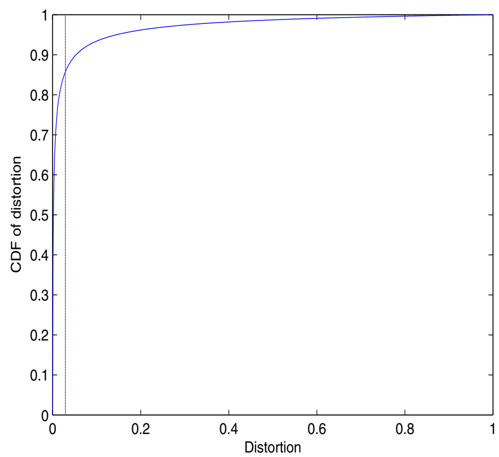
where B is the effective bandwidth and $\bar{\gamma}$ is the average SNR. The CDF for distortion can then be obtained as

$$\begin{aligned} F_d(d) &= Pr\{D \le d\} = Pr\{\sigma^2 2^{-2C} \le d\} \\ &= Pr\{-2C \le \log_2 \frac{d}{\sigma^2}\} = Pr\{C \ge 0.5 \log_2 \frac{\sigma^2}{d}\} \\ &= 1 - F_c(.5 \log_2 \frac{\sigma^2}{d}) = \exp(\frac{1 - 2^{\frac{1}{2B} \log_2 \frac{\sigma^2}{d}}}{\bar{\gamma}}) \\ &= \exp(\frac{1 - (\frac{\sigma^2}{d})^{\frac{1}{2B}}}{\bar{\gamma}}) \end{aligned} \quad (16)$$

In Figs. 5 and 6, we plot the CDF for capacity and source distortion at the receiver for an average SNR of 5 dB and 15 dB, respectively. The vertical lines in the plots



(a) Distribution of capacity



(b) Distribution of distortion

Fig. 6. CDF of capacity and distortion for an average SNR of 15 dB

indicate the mean capacity and mean distortion. Comparing Figs. 5(a) and 6(a), we see that for both SNRs, the mean capacity is achieved with a probability of nearly 0.5, and an increase in SNR increases the mean capacity value. However, considering source distortion at the receiver, as shown in Figs. 5(b) and 6(b), an increase in SNR significantly improves the probability of exceeding the expected distortion. For an SNR of 5 dB, the expected distortion is achieved with a probability of 0.7 and an increase in SNR to 15 dB, not only reduces the overall expected distortion, but also increases the probability of achieving the average to nearly 0.9. The reason for the difference can be understood by investigating the probability density functions (pdfs). The pdf of capacity is roughly symmetric for both the SNRs considered whereas due to the exponentially decaying nature of the exponential function, the pdf of distortion is skewed toward the lower values of distortion.

V. Conclusions

We investigate the classical definitions of channel capacity with receiver CSI and no transmitter CSI for lossy transmission of source information over Rayleigh fading channels. We evaluate the source distortion at the receiver for two definitions of channel capacity, *ergodic capacity* and *outage capacity*, and show that the outage probability required to maximize outage rate is quite different from the outage probability to minimize source distortion at the receiver. We also show that maximizing outage rate has a larger standard deviation of source distortion at the receiver than minimizing expected distortion. We evaluate the source distortion at the receiver for the case when CSI is available at the transmitter and the data rate is adapted to achieve instantaneous capacity. We show that the probability of achieving the mean distortion increases with an increase in SNR, while the probability of achieving the average capacity does not change significantly with SNR. Finally, we propose different optimization criteria that appear more suitable for source transmission over fading channels than ergodic or outage capacity.

References

[1] M. van der Schaar and S. Shankar, "Cross-layer wireless multimedia transmission: Challenges, principles, and new paradigms," *IEEE Wireless Communications Magazine*, vol. 12, no. 4, pp. 50–58, August 2005.

[2] S. Choudhury and J. D. Gibson, "Payload Length and Rate Adaptation for Multimedia Communications in Wireless LANs," *To appear in IEEE Journal on Selected Areas in Communications (special issue on Cross-Layer Optimized Wireless Multimedia Communications)*, 2nd quarter 2007.

[3] K. E. Zachariadis, M. L. Honig, and A. K. Katsaggelos, "Source Fidelity over a Two-Hop Fading Channel," *Proc. MILCOM, International Conference on Military Communications*, pp. 134–139, November 2004.

[4] D. Gunduz and E. Erkip, "Source and channel coding for quasi-static fading channels," *39th Asilomar Conference on Signals, Systems and Computers*, Nov 2005.

[5] J. N. Laneman, E. Martinian, and G. W. Wornell, "Source-channel diversity approaches for multimedia communication," *ISIT 2004*, June-July 2004.

[6] A. Goldsmith, *Wireless Communication*. Cambridge University Press, 2005.

[7] J. Heiskala and J. Terry, *OFDM Wireless LANs: A Theoretical and Practical Guide* . Sams Publishing, December 2001.

[8] T. Berger, *Rate Distortion Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1971.

[9] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communication aspects," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2619–2692, October 1998.

[10] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. McGraw-Hill Science, 2001.

[11] Y. Eisenberg, F. Zhai, and C. E. Luna and T. N. Pappas and R. Berry and A. K. Katsaggelos, "Variance-aware distortion estimation for wireless video communications," *Proc. ICIP*, Sept. 2003.

[12] S. Choudhury, N. Shetty, and J.D. Gibson, "'MOSx and Voice Outage Rate in Wireless Communications," *40th Asilomar Conference on Signals, Systems and Computers*, Oct-Nov 2006.

[13] N. Shetty, S. Choudhury, and J.D. Gibson, "Voice Capacity under Quality Constraints for IEEE 802.11a based WLANs," *International Wireless Communications and Mobile Computing Conference*, July 2006.

[14] J. Hu, S. Choudhury, and J.D. Gibson, "PSNRr,f: Assessment of Delivered AVC/H.264 Video Quality over 802.11a WLANs with Multipath Fading," *Multicomm 06*, June 2006.