Path Diversity and Multiple Descriptions with Rate Dependent Packet Losses

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***Abstract*—Simple path diversity, where the source packet is transmitted simultaneously over multiple paths, is an effective method to combat packet losses or link failures for communications over a wireless network. The bandwidth inefficiency inherent in a path diversity method can be reduced by using multiple description (MD) coding. We consider a scenario where two independent parallel paths/links are available for communicating delay-constrained packetized multimedia data over a wireless network. Motivated by the IEEE 802.11 protocols, often used in wireless LANs and mobile ad hoc networks, we drop a packet at the receiver if even a single bit in the packet is in error. As a result, the packet loss probability is proportional to the packet size. In typical packet-based networks, headers are added to packets and when the packet payloads are small, these headers can dominate the packet size. We compare the average distortion per symbol achieved at the receiver for simple path diversity methods against MD coding of a memoryless Gaussian source for packetized communication over additive white Gaussian noise channels. First, we compare the performance when a packet consists of source information only and next, we show the effect of packet overheads on the performance of each of the methods. With packet headers included, we see that a simple path diversity method requires only a slight increase in transmitted bits and consistently gives lower distortion than the MD method. We also demonstrate the penalty due to MD optimization if headers are neglected.**

I. INTRODUCTION

A. Scenario

We consider a scenario where two independent parallel paths/links with similar channel conditions are available for packetized multimedia communication between two nodes. These parallel links are modeled as independent AWGN channels introducing independent bit errors in the transmitted packets. Motivated by the MAC and physical layer protocols in the IEEE 802.11 standard, often used in wireless LANs and mobile ad hoc networks, we drop a packet at the receiver if even a single bit in the packet is in error. As a result, the packet loss probability is proportional to the packet size. Another characteristic of these protocols is that they add large headers of up to 68 bytes (28 bytes for MAC header and 40 bytes for IP/UDP/RTP headers) to the payloads. When the payloads are small, on the order of tens of bytes, as is typical in conversational voice communications, the packet headers dominate the packet size and hence affect the packet loss rate significantly. The packet efficiency can be improved by increasing the payload size, but because multimedia data is delay constrained and large payloads contribute to end-to-end delay, latency is increased unacceptably.

We compare simple path diversity methods against multiple description coding of a memoryless Gaussian source for packetized communication over additive white Gaussian noise channels. The comparison is based on the average distortion per symbol achieved at the receiver. First, we examine the performance of each of the methods when a packet consists of source information only under different bit error rate (BER) conditions. Next, we show effect of packet overheads on the performance of each of the methods.

B. Prior work

In [1], the authors compare source coding diversity (multiple description coding) and channel coding diversity for on-off and continuous channel models. For on-off channel components, source coding diversity achieves better performance than channel coding diversity because source coding diversity can be used more effectively in adapting the distortion at each description according to the channel failure probability. For AWGN channels with Rayleigh fading, the authors use the distortion exponent that measures how fast the average distortion decreases with an increase in SNR to show that optimal channel coding is more efficient than source coding diversity.

In [2], the authors consider wireline networks with an assumption that no errors are introduced in the packets and compare a two-description coding system against a single description coding system on the basis of the average end-to-end distortion for different levels of congestion in the network. The authors show that the MD coding system performs better than the SD system for high network loading, mainly because of the the smaller packet sizes for the MD system. However, the authors do not consider packet headers, which could significantly affect the capacity and loading in a network when the payloads are small.

In [3], the scenario is again on-off parallel wireless channels. The channel failure is non-ergodic, i.e., independent of the SNR and the encoding rate. The authors consider MD coding with time sharing on the parallel channels and multiresolution coding with space-time coding as two different strategies and compare their performance. They show that MD coding

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performs better than space-time coding with respect to the end-to-end distortion in such cases for a broad range of SNRs.

In [4], the authors consider MD coding for use in sensor networks and investigate the effect of finite buffers and header information on the optimal number of descriptions for transmission over a channel with a probability of packet loss independent of the packet size. They show that the optimal number of descriptions decreases with an increase in the header-payload ratio.

C. Our Work

Our scenario is similar to the on-off parallel channels case but with the probability of failure being proportional to the packet size. In such a scenario, an MD coding method should outperform a single description (SD) method because the packet size for each description in MD is smaller than the SD packets and because MD coding has a higher probability of delivering some information since the probability of packets being dropped on all the independent parallel channels simultaneously is small. Traditionally, MD coding has been compared with SD coding only. However, for realistic wireless networks, there may be additional overheads such as packet headers. In this paper, we consider the effect of packet headers and show that when payloads are small, the gains that can be achieved with MD coding are much less significant compared to simple path diversity methods.

We consider the cases of MD coding with two side descriptions of rate $R/2$ sent over two independent links, an SD code of rate R sent over a single link, an SD code of rate $R/2$ duplicated over the two links, and an SD code of rate R duplicated over the two links. First, we compare the performance of the different methods considered, without considering any overheads, and observe the usual advantages for MD coding. Next, we include packet headers in our analysis and see that the path diversity methods are not much less efficient than MD coding, because the same length headers have to be sent for both MD and path diversity methods and headers dominate the packet size.

II. MULTIPLE DESCRIPTIONS AND PATH DIVERSITY

We compare the following four different methods of communication :

1) **Single description (SD) code of rate** R $(bits/symbol)$ **without path diversity**: For transmitting the single description code, we use only one of the available pair of links (Fig. 2).
2) **Multiple description (MD) coding**: We consider a two-description coder (Fig. 1), where each description is of rate $R/2$ $(bits/symbol)$ and the joint description is of rate R.
3) **Path diversity with rate** $R/2$ $(bits/symbol)$ **SD code**: A single description of the source coded at a rate $R/2$ is duplicated over the two available links.
4) **Path diversity with rate** R $(bits/symbol)$ **SD code**: A single description code of rate R is duplicated over the available pair of links (Fig. 3).

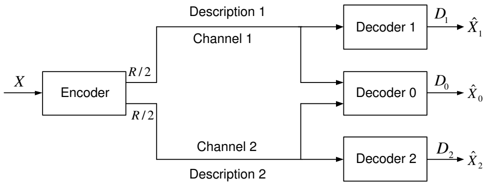


Fig. 1. A two-description coder with each description of rate R/2 sent over two independent channels

Note that the total rate for the first three methods equals R, while the last method has a combined rate of $2R$. This might seem an unfair comparison as a rate R single description code must obviously perform better than a rate $R/2$ code, but, we choose this case because we consider a channel model where the probability of packet loss is proportional to the rate. This case is interesting because it highlights the trade-off between the distortion introduced at the encoder and the distortion due to losses in the channel for common wireless local area network protocols.

Let the source be i.i.d. Gaussian with zero mean and unit variance and the distortion measured by the squared error between the source and the reconstructed sample. The packet loss rate p, when independent bit errors are introduced by the channel, is given by

$$p = 1 - (1 - BER)^L \tag{1}$$

where L is the packet length in bits and BER is the bit error rate. If each packet contains a fixed number N of symbols and each symbol is coded at an average rate of R bits per symbol, the packet length L is now related to R and T as

$$L\ (bits/packet) = R\ (bits/symbol) \times\ N\ (symbols/packet) \tag{2}$$

A. Multiple Description Coding

Figure 1 is an illustration of a two-description coder where the source is coded into two descriptions of rate $R/2$ each and transmitted separately over two independent links. When only description I (II) is received, the distortion is D_1 (D_2), and when both the descriptions reach the receiver, the central decoder reconstructs the source with a distortion D_0. We consider a symmetric coder where each side description is of the same rate and each gives the same fidelity reconstruction of the source.

1) Two Cases of MD Coding:

a) No Excess Marginal Rate: The individual descriptions of rate $R/2$ are rate-distortion optimal with distortion $D_1 = 2^{-R}$ and the lower bound on the distortion for the joint description (D_0) of rate R for a sequence of i.i.d. Gaussian random variables with unit variance and squared-error distortion measure is given by [5]

$$D_0 \geq \frac{2^{-R}}{2 - 2^{-R}}$$

b) No Excess Joint Rate: In this case, the joint description at rate R is rate-distortion optimal with $D_0 = 2^{-2R}$ and

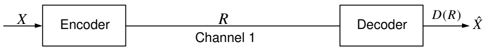


Fig. 2. A single description coder with the coded stream sent over a single channel

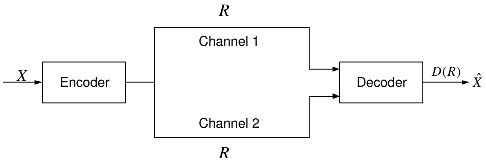


Fig. 3. A single description coder of rate R duplicated over the parallel channels

the lower bound on the distortion at the side decoders for i.i.d. Gaussian sources is given by [5]

$$D_1 \geq \frac{1}{2}(1 + 2^{-R})$$

2) Optimal MD coding: The achievable distortion region for a Gaussian source with unit variance and a fixed rate R ($R/2$ for each description), using MD coding is given by [3]

$$D_1 \geq 2^{-R} \tag{3}$$

$$D_0 \geq 2^{-2R} \tag{4}$$

$$(D_0, D_1) = (a, \frac{1+a}{2} - \frac{1-a}{2}\sqrt{1 - \frac{2^{-2R}}{a}}) \tag{5}$$

$for\ a \in [2^{-2R}, 2^{-R}/(2 - 2^{-R}]$ where D_0 is the distortion at the central decoder and D_1 is the distortion at the side decoders. For a packet loss rate p, the average distortion achieved at the receiver using a two-description coder is

$$D_{MD} = (1-p)^2 D_0 + 2p(1-p)D_1 + p^2 \tag{6}$$

From Eqs. (5) and (6), we get [3]

$$D_{MD} = (1-p)^2 a + 2p(1-p)(\frac{1+a}{2} - \frac{1-a}{2}\sqrt{1 - \frac{2^{-2R}}{a}}) + p^2 \tag{7}$$

For each R and p, we can find the value of 'a' in Eq. (7) that gives the minimum average distortion. This minimum distortion is only achievable when the sender knows *a priori* the packet loss rate and hence can choose the best MD coding method. This gives us a lower bound on the distortion achieved using MD coding but practically achieving this lower bound for changing p's is not possible when information about the channel is not known at the encoder.

For MD coding, we consider three cases: 1) the no-excess joint rate case (MD-NJR), 2) the no-excess marginal rate case (MD-NMR) and 3) the optimal case that gives minimum average distortion for each value of p (MD-OPT).

B. Single Description Coding and Path Diversity

The other three methods of communication we consider (methods 1, 3 ,4 listed in the previous subsection) involve the use of SD coding. Henceforth, we call an SD coder that operates at rate R with an optimal distortion of $D_{FR} = 2^{-2R}$ as the full-rate (FR) coder and an SD coder that operates at $R/2$ with optimal distortion $D_{HR} = 2^{-R}$ as the half-rate (HR) coder.

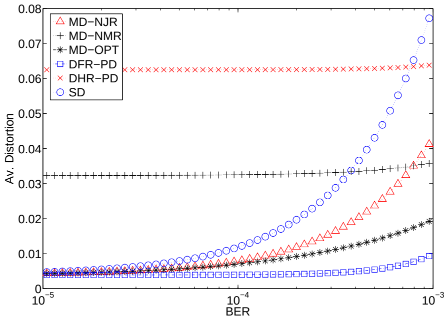


Fig. 4. Average Distortion for the different communication methods considered when source rate is fixed at $R = 4$ and no overheads are added to the packet payload

The average distortion for each of the communication methods that involve an SD coder, with probability of packet loss p is given as follows:
Single description of rate R without path diversity (SD)

$$D_{SD} = (1-p)2^{-2R} + p \tag{8}$$

Half-rate coder with path diversity (DHR-PD)

$$D_{DHR-PD} = (1-p)^2 2^{-R} + 2p(1-p)2^{-R} + p^2 \tag{9}$$

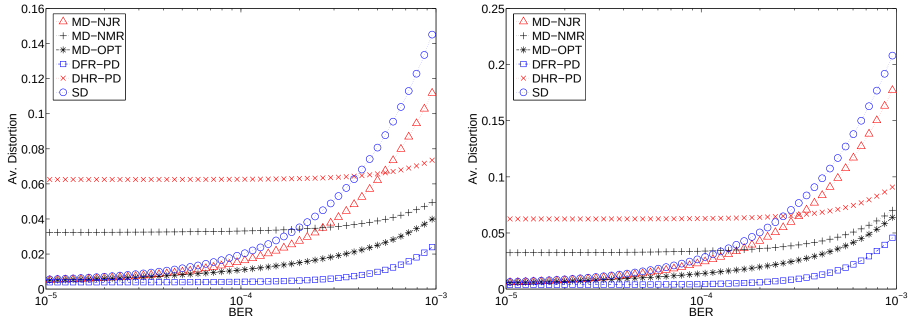
Full-rate coder with path diversity (DFR-PD)

$$D_{DFR-PD} = (1-p)^2 2^{-2R} + 2p(1-p)2^{-2R} + p^2 \tag{10}$$

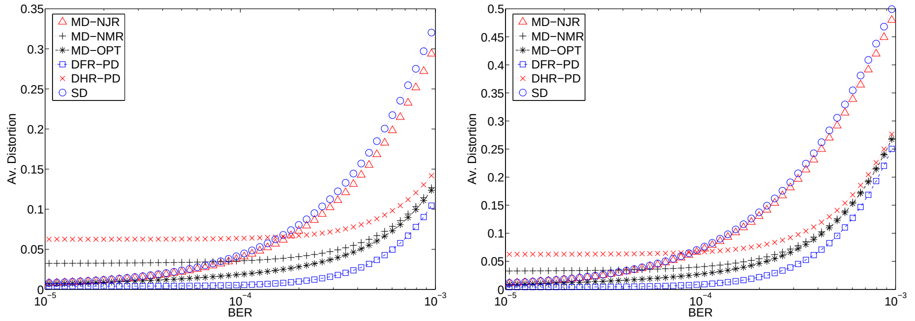
C. Effect of Packet losses

For our analysis, we shall consider a difference in distortion of less than 0.01, i.e. within one percent of the variance, as negligible. We consider rate $R = 4$, since at this rate, the distortion due to encoding $(2^{-2\times 4})$ is less than $.01$ for the full rate coder when there are no losses in the channel. Another important decision for packetization is the number of symbols in each packet. Here, we assume that the number of symbols per packet is limited to 20, resulting in 80 bits per packet. Such packet lengths are common in packet based voice communications.

In Fig. 4, we compare the performance of each of the methods for different bit error rates in the channel. DHR-PD has the worst performance because of the higher distortion at the encoder for the half rate coder and because the average distortion for DHR-PD does not decrease even if both links successfully deliver packets. The remaining methods, SD, MD-NJR, MD-OPT and DFR-PD all give very similar distortions at low BERs. This is because when there are no losses in the channel, SD, MD-NJR and DFR-PD should result in essentially the same distortion due to encoding and MD-OPT should coincide with the no excess joint rate MD case, MD-NJR. Observe that DHR-PD and MD-NMR show very little increase in distortion at high BERs also. This is because these methods have optimal bitstreams on both paths and the distortion at the receiver is almost the same even if only one of the links successfully delivers most of the packets. We see a large deviation in SD, because SD completely fails when the single link carrying the source information fails and the



(a) $L_H = 1 \times NR$ (b) $L_H = 2 \times NR$



(c) $L_H = 4 \times NR$ (d) $L_H = 8 \times NR$

Fig. 5. Average Distortion for different communication methods for high BERs and different header rates. Full rate, $R = 4$. Source rate for each description of MD coders and HR coders is $R/2 = 2$. **NOTE THE DIFFERENCE IN SCALES FOR AVERAGE DISTORTION IN (a) -(d)**

distortion at the receiver is maximum. Similarly, for MD-NJR, if one of the links fails, then the distortion increases considerably because the side descriptions are not optimal. DFR-PD also shows only a small deviation because of path diversity.

III. PACKET HEADERS

In packet based networks, headers are added to each source packet by other protocol layers to facilitate communication over the network. Such overheads affect the probability of packet loss, p, as they increase the length of the packet. If the number of header bits per packet is L_H , then p is given by

$$p = 1 - (1 - BER)^{NR+L_H} \tag{11}$$

We now investigate the behavior of each of the methods for different values of L_H. We consider different payload-header ratios ($NR : L_H$) $1 : 1$, $1 : 2$, $1 : 4$, $1 : 8$. Such ratios typically occur in voice communications over IEEE 802.11 based WLANs. For example, each packet sent in the transmission of G.729 [6] encoded speech contains 10 (10 ms of speech) or 20 bytes (20 ms of speech) of payload and around 68 bytes of overhead. Speech encoded with AMR-WB [7] at 12.65 kbps contains 32 (20 ms of speech) bytes of payload and 68 bytes of overhead per packet and a G.711 [8] packet contains 80 (10 ms of speech) or 160 bytes (20 ms of speech) of payload and 68 bytes of overhead.

A. Effect of headers on packet losses and distortion

In Fig. 5 we plot the average distortion curves for different header sizes. Observe that increasing header sizes worsen the performance of all the methods. MD-NMR outperforms MD-NJR at high BERs as expected, because at high loss rates, only one of the descriptions reaches the receiver for a majority of the time and individual descriptions are optimal in MD-NMR. The performance of MD-OPT approaches that of MD-NMR as the BER increases for high values of L_H. This is because, as L_H increases, p increases and only one of the descriptions reaches the destination most of the time. With only one description reaching the receiver, the best distortion MD can achieve is $D = 2^{-R}$ and this is exactly what each description of MD-NMR achieves.

Although p for DFR-PD is larger than that of all the other methods except SD, because of a higher rate on each link ($NR + L_H$ against $NR/2 + L_H$ for other methods), its distortion is smaller than any of the other methods. The gain due to a higher source rate and path diversity for DFR-PD is large enough to overcome a higher packet loss rate p. If we compare SD and DFR-PD, there is a difference of about 0.25 in the average distortion at the highest BER for $L_H = 8NR$, and all of this gain for DFR-PD can be attributed to path diversity. The most significant point to note here is that after these large headers are added, the effective rate of DFR-PD ($2 \times (NR + L_H)$) differs from any MD method or DHR-PD ($NR + 2 \times L_H$) by only R and when $L_H = 8NR$, DFR-PD requires less than a 6% increase in bandwidth compared to the MD and DHR-PD methods. For such a small increase in bandwidth, the gain in quality achieved using DFR-PD is quite significant.

IV. SIGNIFICANCE OF HEADERS

In this section we show the significance of headers by illustrating the offset seen in the operating point of an MD coder in its achievable distortion region for different header sizes, when headers are not considered in the design.

In Fig. 6 we show the achievable distortion region of an MD coder with each description at rate $R/2 = 2$. The curve represents the lower bound on points (D_0, D_1). Suppose we know that the BER in the channel is 10^{-4}, then we would design the MD coder to operate at a point on the curve that produces the minimum average distortion for the specified BER. The point marked as '*' shows the optimal operating point estimated without considering the headers using $p = 1-(1-10^{-4})^{NR}$. The points marked as 'Δ' and 'O', show operating points determined using $p = 1-(1-10^{-4})^{NR+NR}$ (for $L_H = NR$) and $p = 1-(1-10^{-4})^{NR+8NR}$ respectively in Fig. 6 for $N = 20$ and $R = 4$. The operating points determined after including headers are offset from '*' by a significant amount and the offset increases with the header size. Observe that as L_H increases, the side distortion D_1 at the optimal points decreases while D_0 increases, because a larger L_H implies a larger p and since only one description is delivered most of the time, a smaller D_1 reduces the average distortion. Also, as L_H increases, the minimum average distortion achievable increases as seen from the values of 'D' in Fig. 6.

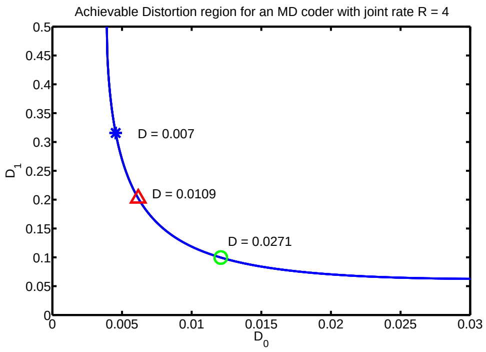


Fig. 6. The curve is the lower bound of the achievable distortion region for a two-description coder with each description of rate $R/2 = 2$. The '*' is the optimal point which gives minimum average distortion for a BER = 10^{-4} when the packet headers are not considered. The optimal point when headers are considered is marked by 'Δ' and 'O' for $L_H = NR$ and $L_H = 8NR$ respectively.

Now, if we do not consider the header bits in the design when actually packet headers of significant size are transmitted, then the probability of packet loss is incorrectly estimated as

$$p_{est} = 1 - (1 - BER)^{NR} \tag{12}$$

We would also expect the distortion at the receiver to be

$$D_{est} = (1 - p_{est})^2 D_0^* + 2p_{est}(1 - p_{est})D_1^* + p_{est}^2 \tag{13}$$

where (D_0^*, D_1^*) is the optimal point '*' on the curve in the Fig. 6. However, the actual distortion observed at the receiver is higher because p is larger than the estimated value due to the additional header bits sent. The actual p, p_{act} is given by

$$p_{act} = 1 - (1 - BER)^{NR+L_H} \tag{14}$$

and the actual distortion observed at the receiver is

$$D_{act} = (1 - p_{act})^2 D_0^* + 2p_{act}(1 - p_{act})D_1^* + p_{act}^2 \tag{15}$$

D_{act} cannot be smaller than D_{est}. In Table I we list, for different header-payload ratios, the D_{act} values (average distortion achieved when the MD coder is designed without the headers taken into consideration, i.e. $L_H = 0$) and the D_{min} values (minimum average distortion that can be achieved when the MD coder is designed with the headers taken into consideration). Observe the large difference in D_{min} and D_{act} values, demonstrating the significance of considering headers in analysis and design of MD coders.

TABLE I
D_{min} AND D_{act} FOR DIFFERENT HEADER SIZES

$L_H : NR$	D_{act}	D_{min}	$\% \ difference$
0	0.0070	0.0070	0
1	0.0143	0.0109	31.19
8	0.0471	0.0271	73.80

V. CONCLUSIONS

We consider multimedia communication over a pair of independent links with rate dependent packet losses. We show that when the payloads are small, as is typical in conversational voice communication, the packet headers can dominate the packet size and the improvement in bandwidth efficiency achieved through MD coding is almost insignificant. The headers dominate the packet size and smaller payloads achieved through MD coding do not reduce the packet loss rate significantly. The no excess joint rate case of MD coding is more useful for low BER conditions and the no excess marginal rate case is more suitable for high BER conditions, but the simple path diversity method of duplicating the full rate coder over both links gives consistently better performance than any of the multiple description methods at the cost of only a small percentage increase in the bits transmitted per symbol. We also demonstrate the importance of considering headers in optimizing an MD coder through the large deviation observed in the average distortion, when headers are included in the optimization and when they are not.

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